IDENTIFICATION OF CRACKS IN BEAMS USING MODAL PARAMETERS
IDENTIFICATION OF CRACKS IN BEAMS USING MODAL PARAMETERS

Thesis presented as a partial requirement to obtain a Master’s degree in Mechanical Engineering, in the Post-Graduate Course in Mechanical Engineering, Technology Sector, Federal University of Paraná.

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ABSTRACT

In the last years, numerous works were developed with the objective of identifying cracks in structures. In them, several numerical models were proposed for the structure with and without cracks, with different optimization techniques, adopting various experimental procedures and monitoring different modal parameters. Given all these possibilities, in this work, some of the proposed methods were reviewed, comparing them and obtaining the advantages and disadvantages of each one. Five different methodologies for identification of cracks were studied. The first one was to apply the wavelet transform on the vibration modes, obtained experimentally, and to verify how the coefficient curves, or the derivative of them, vary. In the second method, it was applied an equation that combines natural frequencies and vibrate modes, to monitor the located flexibilization in the structure. In the third methodology, a graphical approach was applied to obtain the solution of the inverse problem using the natural frequencies as the monitored parameter. In the fourth methodology, it was also used the frequencies, but, instead of the graphical approach, it was applied optimization techniques in the resolution. Finally, in the last methodology, it was applied the previous formulation, but on the vibration modes. In addition to review the methodologies, the work relied on two different numerical models, one built on MATLAB® software and another on ANSYS®. Three optimization techniques were also tested: genetic algorithms (GA), nonlinear optimization techniques (NLOT) and hybrid algorithms. To test the methodologies implemented, two beams with rectangular section were tested, each having a crack, but in different positions. For each crack size adopted, the modal parameters were obtained through experimental modal analysis (EMA) and operational modal analysis (OMA). From the results, it was concluded that the method with graphical approach presented better accuracy with low computational cost. When the vibration modes were used with the optimization algorithm, the errors were bigger than 10% in both depth and location identifications. The method that uses natural frequency with optimization techniques had similar or worst accuracy than the graphical approach. Besides that, the application of wavelet transform on the modes provided the position with the lowest computational time, but did not obtain the fissure depth. Moreover, the method that combine natural frequency and vibration modes did not work.

Key-words: Identification of cracks, Operational Modal Analysis (OMA), Wavelet Transform (WT), Genetic Algorithms (GA).
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<td>Frequency Response Function.</td>
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<td>FEM</td>
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<td>GA</td>
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<tr>
<td>SQP</td>
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<tr>
<td>NLP</td>
<td>Non-Linear Problem</td>
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<tr>
<td>QP</td>
<td>Quadratic Programming</td>
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<td>DFO</td>
<td>Derivative-Free Optimization</td>
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<td>WT</td>
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<td>SSI</td>
<td>Stochastic Subspace Identification</td>
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<td>EFDD</td>
<td>Enhanced Frequency Domain Decomposition</td>
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SYMBOL LIST

Roman alphabet

\( a \) - Depth or size of the crack
\( a_i \) - Identified crack depth
\( b \) - Width of the beam
\( c_i \) - \( i \)-th modal damping
\( C \) - Damping matrix
\( E \) - Elasticity modulus
\( E_a \) - Error in the crack depth
\( E_x \) - Error in the position crack
\( f(t) \) - Vector of acting forces
\( f_i \) - Force acting in the \( i \)-th node
\( g \) - Error function auxiliary
\( G \) - Shear modulus
\( h \) - Height of the beam
\( h_e \) - Size of the element of the FEM
\( I \) - The moment of inertia of the second order of area
\( k_i \) - \( i \)-th modal stiffness
\( K \) - Stiffness matrix
\( K_e \) - Local stiffness matrix
\( K_t \) - Stiffness of the crack
\( L \) - Length of the beam
\( m_i \) - \( i \)-th modal mass
\( M \) - Mass matrix
\( M_e \) - Local mass matrix
\( q(t) \) - Displacement vector
\( \dot{q}(t) \) - Velocity vector
\( \ddot{q}(t) \) - Acceleration vector
\( r \) - Objective function or error function
\( s_j \) - \( j \)-th root of the characteristic polynomial
\( S \) - Cross-sectional area
\( u_1, u_2 \) - Nodal displacement
\( V \) - Volume of the domain
\( V_i \) - Volume of the \( i \)-th subdomain
\( X_t \) - Crack position
\( X_i \) - Identified crack position

**Greek alphabet**

\( \beta \) - Scaling function
\( \gamma \) - Wavelet function
\( \theta_1, \theta_2 \) - Nodal rotation
\( \vartheta \) - Shrinkage coefficient of the Nelder-Mead Method
\( \kappa \) - Contraction coefficient of the Nelder-Mead Method
\( \lambda_j \) - \( j \)-th eigenvalue
\( \mu \) - Reflection coefficient of the Nelder-Mead Method
\( \nu \) - Poisson Coefficient
\( \rho \) - Density
\( \tau \) - Rotation from the shear
\( \phi \) - Normalized vibrate mode
\( \chi \) - Expansion coefficient of the Nelder-Mead Method
\( \Omega_j, \omega_j \) - \( j \)-th natural frequency
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1 INTRODUCTION

The concern with the structural integrity of machines, structures and systems is not new and the countless accidents occurred that justify this interest. Many of them caused not only financial losses, but also decimated lives. (OLIVEIRA, 2017).

Among those incidents, they may be cited those involving the Liberty ships. Of the 4694 manufactured warships, 1289 suffered a brittle fracture or a failure in the welded structures (ALMEIDA, 2017). Due to this, the connection between the fractures, the cracks in the material and the mechanisms of their propagation began being observed.

After these episodes, many other accidents occurred, such as the fractures on Comet 1 aircrafts in 1954, the fall of the Pleasante bridge in Virginia in 1967 and the accident with the Flight 423 of the Aloha Airlines in 1988 (ALMEIDA, 2017). All of these failures were caused by the propagation of cracks in the structure, raising the necessity of the creation of models to identify these cracks and predict their growth.

In order to avoid the catastrophic failure caused by these cracks, four different types of maintenance have been developed over the years, such as: corrective maintenance, preventive maintenance, predictive maintenance and prognostic maintenance. The latter is the most recent and advanced and it consists of continuously monitoring some parameters of the structure in order to predict the remaining useful life (OLIVEIRA, 2017).

For to make this prediction, it is necessary that the crack be reliably identified. Numerous techniques can be used for this, such as the use of penetrating liquids, ultrasonic measurements, acoustic emissions and others. Among these procedures, methods based on vibration response measurements have received special attention, because they monitor the variations of the dynamic characteristics of the structure.

About the structures studied within the engineering, the beams can be detached. By definition, these structural elements are only subject to transverse loading. Due to the simplicity, there are numerous applications, modeling and analytical and numerical formulations for their behavior. Another point is that, as
the beams are present in several complex systems, the identification of cracks in them becomes necessary.

Numerous are the works developed both in the modeling of the crack and in the identification of the parameters of it, as can be evidenced by the extensive bibliography used. From these works, it is possible to notice that different modal characteristics were monitored, various optimization techniques were used, numerical models for the intact and cracked beam were built and methods of identification of the crack were studied. Due to the great variety of methodologies used, this dissertation will focus on obtaining the parameters of the crack through natural frequencies and modes of vibration.

The present work will have an experimental stage to survey the modal parameters of a metal beam with conditions of free-free contour. This data acquisition will be performed by experimental modal analysis and operational modal analysis, also seeking to compare the data obtained by these two techniques. On the obtained data will be applied the methodologies of identification chosen, which will be better explained in the section of Materials and Methods.

The numerical models used will be built inside the MATLAB® software, according to the proposal of FEM for beam with formulation of Timoshenko and Euler-Bernoulli, and within commercial software ANSYS®. The optimization techniques used, when necessary, will be genetic algorithms (GA), Sequential Quadratic Programming (SQP) and hybrid algorithm between GA and Nelder-Mead method (or SQP, in some cases). The numerical models and the optimization techniques used will be better explained in the Theoretical Foundation chapter.

Once all the proposed cracks have been identified, the methods used will be compared, addressing the advantages and disadvantages of each of them. This comparison will include the acquisition techniques used, the numerical models applied, the optimization methods chosen and the identification methodology implemented.

1.1 JUSTIFICATION
Numerous work has been developed in the area of crack identification, varying models for intact and cracked structure, modal parameter used, optimization technique and experimental procedure adopted. For the description of the structural behavior of the cracked component, numerous models were developed, some of them being by finite element method, finite element method with applied wavelet, continuum mechanics among other consecrated models. The crack, in some works, was treated as a torsional spring, while, in others, it was considered as an element of reduced stiffness. It is worth mentioning that some studies have developed elements with their own formulation to describe the function of the fissure.

Regarding the input parameters used, there are methodologies that only use variations in the natural frequencies, while others monitor changes in the component's vibration modes. In addition, there are still those that combine the modal parameters in the identification. In the optimization technique, the minimization of the objective functions can be performed using evolutionary algorithms, nonlinear optimization techniques or even hybrids of them.

Different combinations of the mentioned tools can be used to solve the inverse problem. However, no papers compare and revise these techniques for the same experiment. Therefore, it is difficult to determine which methodology identifies cracks with better precision. In addition, no research discuss the advantages and disadvantages of each procedure.

1.2 GOAL

1.2.1 General goal

The main objective of the work is to perform a review of existing methods for identification of a crack in a beam using modal parameters, such as natural frequencies and vibration modes.

The criteria that will be evaluated are the precision and the computational time spent in the process. In addition, the advantages and disadvantages of using each of them will be raised. It is sought with this review of methods to unite in a single work the different techniques already developed.
1.2.2 Specific goals

Besides of the general objective, some specific objectives were defined in order to better guide this review. In a simple and brief way, they consist of:

a) Use operational modal analysis (OMA) and experimental modal analysis (EMA) in the data acquisition in order to compare how each experimental technique impacts the identification;

b) Construct numerical-computational models using the finite element method for the structure within MATLAB® and ANSYS®, in order to verify the influence of the numerical model on the identification;

c) Develop optimization algorithms for the identification of crack using three different methods to obtain the solution of the problem. This will allow to evaluate the performance of these three techniques in the identification of cracks in single beams;

d) Compare the precision and computational time of each combination of data acquisition technique, modal parameter(s), numerical model and optimization method, discussing the advantages and disadvantages of each one;
2 BIBLIOGRAPHIC REVIEW

According to Dimarogonas (1996), the presence of cracks in the structure generate a localized flexibilization, influencing the dynamic and vibrational behavior. From this, numerous works were developed in order to model how the cracked structures behave and how the cracks influence the modal parameters.

In 1990, Rizos identified the position and depth of a crack in a crimped-free beam by modeling the fissure as a torsional spring and the remainder of the beam through continuum mechanics. Newton-Raphson method and variations in natural frequency were used to solve the optimization problem. According to Rizos, this technique showed good results, but they were not superior to those obtained by ultrasound and radiography. In addition, since small cracks induce slight variations in the frequencies, fissures of less than 10% of beam height were not addressed.

Montalvão et al. (1990) built a database with the natural frequencies of beams with cracks in different positions and depths. The purpose of the paper was to provide data for that other researchers could validate the identification techniques created. It is worth mentioning that the crack was modeled as a fine cut, not adding nonlinearities caused by the closing of it. Studies were done on how thin the cut thickness should be so that the dynamic behavior was similar to that of a real crack, and, after that, the database of the first four natural frequencies was created.

Hu and Liang (1993) sought to propose a crack identification methodology that did not use optimization techniques. In the work, the structure was first modeled with finite elements and the crack was considered to be a torsional spring, similar to that realized by Rizos et al. (1990). The difference in the proposed identification was the use of graphical overlap to obtain fault parameters. To demonstrate this technique, the methodology was applied in a numerical model of a bi-supported beam with two fissures. The positions and depths were varied so that, for each mode, a surface was obtained for the variation of the natural frequency between the beam with and without cracks. After this step, the differences between natural frequencies of the structure with the cracks that were to be identified were verified. Having these values, the surfaces of each mode were sectioned in the level of these variations allowing
obtaining curves, for each mode. These curves were overlapped and where they intersected was obtained the position of the crack and the stiffness of a spring equivalent. This stiffness could be converted to the depth of the crack without great difficulties. The proposed methodology presented good results, but raised the need for more studies, working with very complex structures.

Narkis (1994) applied again the methodology proposed by Rizos et al. (1990) and Ostachowicz and Krawczuk (1991) in the identification of cracks. Moreover, it was discussed that to obtain the position and depth is necessary, at least, two natural frequencies. This occurs, because different combinations of these parameters can generate the same variation in a single natural frequency.

In Chondros et al. (1998) were modeled cracked beams using continuum mechanics, as was done by Rizos et al. (1990). However, for the crack modeling, a disturbance function was used in the equation of motion. Experiments were performed and, through the developed technique, it was possible to identify cracks with depths of 10% to 60% of the height of the beam. For greater depths, the modal coupling made a reliable measurement difficult and, for smaller cracks, the frequency variation was very small.

Unlike previous works, Lu and Hsu (2002) proposed the use of wavelet transform (WT) to identify crack in strings, with the cracks being simulated as springs and masses trapped in the intact structure. The WT was applied on the displacement vector of the deformed structure at time. It was possible to notice a distortion in the coefficients curves of the WT in the region where the simulated fissure were. This disturb could be an alternative to identify the location of the crack.

For identification of the crack position, Cacciola et al. (2003) proposed a statistical analysis of the structural response caused by the application of white noise. As a numerical model, the beam was modeled using finite elements with Euler-Bernoulli formulation. In the element in which the crack was located, the stiffness was reduced in order to increase the flexibility of the region. The method failed to obtain the location of very small cracks and was unable to estimate the depth of failure.

Following Hu and Liang’s (1993) approach, Patil and Maiti (2003) studied the identification of multiple open cracks in a beam from natural frequency measurements. In this work, the intact regions were modeled by the continuum...
mechanics, according to the Euler-Bernoulli formulation, whereas, for the cracks, torsional springs were used. The proposed model did not cover beams with considerations of rotation, significant damping and shear effects. In order to obtain the natural frequencies, the structure was segmented and an iterative process was implemented, as described by Mahmoud et al. (1999).

Patil and Maiti (2003) used, as in previous works, natural frequency measurements to identify two and three cracks on a shaft. In the numerical model, the position of the crack and stiffness of the torsion spring were varied, so that the change in natural frequency was the same as experimentally obtained. This allowed obtaining curves that related these parameters. The fact that different pairs of position and depth allow to obtain the same variation in a natural frequency was addressed, as such in (Rizos, 1990), (Ostachowicz and Krawczuk, 1991) and (Narkis, 1994). The obtained curves for each vibration mode were overlapped and the intersection allowed determining the failure parameters. This methodology followed what Hu and Liang (1993) proposed.

Based on the idea that the fissure generates a localized flexibilization, a method was proposed in the book Maschinendynamik of Dresig and Holzweißig (2006), which relates the vibration modes of the intact structure to the variations of the natural frequencies. The differential of this method is that a numerical model for the crack is not necessary, only the experimental data and the application of an optimization method are required.

Chasalevris and Papadopoulous (2006) dealt with the identification of multiple cracks in shafts, but with the differential of them having an angle of lag in their orientation. That is, the faces of the fissure are parallel, but the roots of them have a certain angulation. As in other works, the cracks were shaped like springs with rigidities dependent on depth and angulation. The intact portion of the structure was modeled via the continuum mechanics with the Euler-Bernoulli formulation. The proposed technique was divided into two stages. In the first one, the WT was applied to the vibration modes, in order to obtain the location of the fissures. The second part consisted of overlapping the frequency difference curves for each mode, in a technique similar to that approached by Patil and Maiti (2003). The difference for this case was that the curves related depth and angle between the cracks that generated the same difference in the natural frequency.
Xiang et al. (2006) solved the direct and inverse problem for a cracked beam using a finite element model with B-spline wavelet applied in the interval (Finite Element Method of a B-Spline wavelet on the Interval) for intact region and torsion spring for the crack. As was done by Patil and Maiti (2003), the curves that correlate position and depth for a given natural frequency difference were superimposed and the crack was identified. The validation of the technique for the inverse problem was performed numerically and experimentally. In the numerical validation, the natural frequencies of the structure without fissures and the structure with a known crack were obtained. This difference between the frequencies was considered the input parameter to obtain the curves that related position and depth. With the intersection, it obtained the position and depth that coincided with the fissure parameters chosen previously. In the experimental verification, the natural frequencies of the cracked and intact beam were obtained by the impact test. With these values, the curves of position versus depth were obtained, overlapped and the crack parameters were identified that should coincide with the actual values of the fissure. The proposed methodology presented a good performance in the numerical validation, but not in the experimental one.

As with Chasalevris and Papadopoulos (2006), Presezniak (2007) applied WT on the vibration modes of a cracked beam with free-free contour conditions to identify the crack location. The work had an experimental stage to validate the methodology. The obtained results were satisfactory results. However, many points were needed to construct the modes, making the technique not feasible for complex, two-dimensional or large structures.

Repeating the procedure of Xiang et al. (2006), Xiang et al. (2007) worked with the identification of cracks in a rotor shaft. The numerical models used for the intact structure and for the fissure were the same ones applied in the previous work. The differential was the addition of discs in the shaft, which were modeled by FEM BSWI, but with formulation of the Timoshenko. The graphic overlay was used in the identification too.

Xiang et al. (2008), as in previous work, used FEM BSWI for modeling the intact structure and torsion spring for the crack. Instead of using the graphical method to identify the crack, as in previous works, an optimization was performed, using genetic algorithms to obtain the depth and location. In
conclusion, the hypothesis of the necessity of the development of methods that use operational modal analysis in the identification was raised, so that it was possible during the operation of the component.

As an alternative to graphic overlapping, Rosales et al. (2009) compared the identification using graphical power curve method in the identification with the application of neural networks. The latter method presented greater errors than the one that used the power curves. However, neural networks have the advantage of being more generic and not depending on the numerical model used for the structure. This lower accuracy is due to the necessity of an extensive network training with an experimental database.

Zhang et al. (2009) applied a mixed approach to solve the inverse problem for a staggered beam with cracks. To identify the position, it was applied the WT and, to find the depth, it was used the transfer matrix. In it, the position of each crack and the natural frequencies was used as input and the flexibilization curves were obtained. The intersection of them corresponded to the depths of each of the cracks. This work did not have an experimental stage for validation of the technique, and, to provide the input parameters for the method, it was used ANSYS®.

Using a methodology similar to that of Chasalevris and Papadopoulos (2006) and Zhang et al. (2009), Xiang and Liang (2012) applied WT on the vibration modes to obtain the crack location and used the variation in the natural frequencies to identify the depth. In this second, the graphical overlapping methodology of the curves was applied, but, in this work, the curves related the depth of the two cracks, since the positions were known. According to Xiang and Liang, the vibration modes have a higher sensitivity than the natural frequency for crack detection and this efficiency is amplified by WT. To validate the method, an impact test was performed to survey the modal parameters of a beam with two cracks present.

Disregarding the variation of the vibrate mode due to failure, Behzad et al. (2013) treated the identification of cracks with a 90° angle between their roots. That is, one crack was positioned on the upper face of the beam while the other one was on the side face. The formulation proposed by Bezhad converged to a nonlinear system whose solution was obtained thanks to the ‘fsolve’ tool of MATLAB. This tool uses a combination of the Levenberg-Marquardt method,
region of confidence and a variant of the Powell method. The proposed methodology presented a good performance, having errors less or equal to 6%, besides allowing the application to an indefinite number of cracks. The only problem was that the method is not able to identify cracks of different orientations, but in the same position.

In order to improve the vibration mode data obtained experimentally, Cao et al. (2014) proposed the use of the Teager power operator to reduce the influence of noise on the measurement. After that, the wavelet transform was applied on the vibration modes increasing the capacity to identify the discontinuities. For the numerical study, the intact beam region was considered as Euler-Bernoulli beam member and the crack was modeled as a torsional spring. In addition to the numerical part, an experimental stage was carried out to evaluate the methodology. The results were satisfactory, according to Cao et al. (2014).

Masoumi and Ashory (2014) performed a multi-resolution analysis of WT of the vibration modes, in order to detect the crack in a beam. For this analysis, it was proposed to interpolate the data, so that the curves presented more points, allowing the application of a larger number of filters. In this work, two different wavelets (family of Symlets and biortogonals) were evaluated and the peculiarities of each one were compared. In addition, the application of continuous wavelet was studied, in order to improve the efficiency of the technique. There was a step of identifying the position of the fissure in vibrate modes. These modes were obtained by a numerical model of a cracked beam. In this stage, the location errors oscillated from 1 to 10%, varying with position, depth, wavelet type and the presence of post treatment data after WT application. After this step, an experimental part was performed and, on the mode data, the decomposition was done in multiple filters. The objective was to demonstrate that the higher the number of filters, the more prominent the effect caused by the crack. However, no localization was identified on these experimental data.

Different from the works presented, Zhou et al. (2015) used the transmissibility curve for crack detection, as well as the operational modal analysis for the curve acquisition. However, no steps were taken to identify the crack, adding no further novelty in the field of study.
Hou and Lu (2016) solved the inverse problem using finite element method (FEM) for numerical model and genetic algorithms in the optimization process. However, the objective function combined the relative differences between the natural and experimental natural frequencies with the differences between the numerical and experimental normalized vibration modes. In addition, the mesh of FEM was adaptive, so that the crack was closest to the center of the element, ensuring that its influence was only on the elements in which it was located. This improvement, together with the application of AG and differentiated mesh element, ensured a high performance identification methodology with a small number of elements.

As discussed by Xiang and Liang (2012), the vibrate modes are more sensitive to the presence of cracks than the natural frequencies. Therefore, Oliveira et al. (2017) used their variation to identify the fissure. In this work, a study was made about the impact of the fissure in the vibration mode in different points of the beam. This allowed to optimize the number of points needed for reliable identification. Besides this study, an experimental stage was carried out to validate the method. In this phase, the vibration modes were collected via OMA, only at one measurement point, which differed for each analyzed natural frequency. Four natural frequencies were monitored, and the collection coordinate for the third and fifth modes was the same. On these data, it applied the identification method that had genetic algorithms as a tool optimization. The methodology was able to characterize cracks with depth less than 10% of the height of the beam, presenting errors less than 10%.

Still in the field of crack identification, Eroglu and Tufekci (2016) worked with natural frequencies and MEF with rotation and shear considerations, Corrêa et al. (2016) with plaque identification by means of a particle swarm algorithm of a cohesion parameter of the structure flexibility network. In addition, Mostafa and Tawfik (2016) proposed to combine axial and flexural frequencies to identify the crack, Oliveira (2017) with identification of cracks from natural frequency variations using ANSYS® software for numerical model construction. Involving wavelets, there also has Zhang et al. (2016) with the finite element model of multivariate wavelets. Janeliukstis et al. (2017) with the application of WT on vibration modes to identify multiple cracks, in a work similar to that of Zhang et
al. (2009), but with the difference of not performing the identification by the frequency and of the fissures cannot be considered cracks due to the width.

Other important works that deal with crack modeling are those by Qian et al. (1990) that dealt with the crack opening and closing effect. Kisa et al. (1998) developed a methodology that integrates the linear characteristics of the structure with the non-linear ones of the face of the fissure. Bovsunovsky and Marveev (2000) worked with the analytical model of closed cracks, Friswell and Penny (2002) revised the methodologies that describe the behavior of cracks applied to the monitoring of structural integrity in low frequencies, and Khan and Parhi (2013) studied the impact of crack depth on vibration modes and natural frequencies.

As shown, numerous works have been developed on the modeling of the crack and on the identification of the parameters of it. Analyzing them, it is possible to notice that different modal characteristics were monitored, it were used various optimization techniques; numerical models for intact and cracked beam and identification methods. Due to the great variety of methodologies used, this dissertation will focus on obtaining the parameters of the crack through natural frequencies and modes of vibration.

The dissertation will have an experimental stage to survey the modal parameters of a metal beam with free-free contour conditions. This data acquisition will be performed by operational modal analysis and experimental modal analysis, also seeking to compare the data obtained by these two techniques. On the obtained data, it will be applied the methodologies of identification chosen, which will be better explained in the section of Materials and Methods.

The numerical models used will be built inside the software MATLAB®, according to the proposal of FEM for beam with formulation of Timoshenko and Euler-Bernoulli, and within commercial software ANSYS®. The optimization techniques used, when necessary, will be genetic algorithms (GA), Sequential Quadratic Programming (SQP) and hybrid algorithm between GA and Nelder-Mead method. The operation of the numerical models and the optimization techniques used are better explained in the next section.

Once all the proposed cracks have been identified, the methods used will be compared, addressing the advantages and disadvantages of each of them.
This comparison will include the acquisition techniques used, the numerical models applied, the optimization methods chosen and the identification methodology implemented.
3 THEORETICAL FOUNDATION

As seen in the literature review, there are many ways to identify the crack. Before presenting the techniques that will be reviewed and studied, it is important a brief presentation of some theoretical concepts that will be used in the course of the work.

This section will quickly and succinctly discuss the optimization techniques used, how to obtain the modal parameters in systems with multiple degrees of freedom, the application of finite element method in computational models, among other topics. It is worth mentioning that the function of this part of the work is only to provide a theoretical basis of the tools that will be used. It is not intended to replace specific literatures on these subjects. If there is any familiarity with the topics covered, the deletion of this section in reading will not hinder the understanding of the work.

3.1 VIBRATION ANALYSIS OF MECHANICAL SYSTEMS

The presence of cracks flexibilizes the structure locally, which influences the vibratory response (DIMAROGONAS, 1996). That is, natural frequencies, vibrate modes and frequency response functions (FRF) are affected by the fissures. Thus, it is important, for the crack identification, the knowledge of how to model these structures reliably to extract the modal parameters of them.

The components studied in this work are beams and they can be modeled as non-rotating multiple degrees of freedom systems, without significant loss of results. Therefore, the mechanical behavior can be characterized, in general, by the equation (EWINS, 2000):

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = f(t)$$  \hspace{1cm} (1)

where $M$, $C$ and $K$ are the mass, damping and stiffness matrixes, respectively, while $\ddot{q}(t)$, $\dot{q}(t)$ and $q(t)$ are the acceleration, velocity and displacement vectors. The component $f(t)$ is the vector of acting forces.

Analyzing the equations of motion, it realizes that they are composed of a set of $n$ coupled second order differential equations, being $n$ the number of
degrees of freedom of the system. As this value increases, the more complex the solution becomes. In order to facilitate the resolution of this system, a method known as modal analysis can be applied. In this technique, the displacement vector of the masses is expressed as the linear combination of the normal vibration modes of the system. This linear transformation decouples the motion equations, in order to obtain a set of \( n \) non-coupled second order differential equations (RAO, 2008).

In other words, this linear transformation takes the equations of the time domain into the frequency domain, that is, the modal space. In it, the equations are decoupled, and each of them describes the movement of a degree of freedom.

Different models of the system lead to different formulations. Among the models that can be used, they highlight the non-damped, the viscous and the hysteretic models (EWINS, 2000). In the next section, the non-damped model and the solution of the eigenvalues and eigenvectors problem will be presented in more detail. If further study is of interest, it is recommended to read a bibliography focused on the subject, such as Ewins (2000) and Rao (2008).

3.1.1 Eigenvalue problem of an undamped system

In the model of an undamped system of multiple degrees of freedom, the damping matrix \( C \) is considered null. This reduces the equation (1) to:

\[
M\ddot{q}(t) + Kq(t) = f(t)
\]  

(2)

Considering the case of absence of external excitation \( f(t) = 0 \), free vibration) and that the response is given in the form \( q(t) = \phi e^{st} \) has:

\[
(s^2M + K)\phi e^{st} = 0
\]  

(3)

Since \( e^{st} \) is never null, it has:

\[
(s^2M + K)\phi = 0
\]  

(4)
This equation consists of the undamped eigenvalue problem, which, although simplified, presents good results for the modes of vibration, since the damping acts on the amplitude and not on the form of them. To obtain the roots of the characteristic polynomial, it is enough to calculate the determinant of equation (4) and equalize it to zero. Once the roots are obtained, it is observed that they occur in pairs, being pure and conjugate imaginary (Espindola, 1986), as shown in equations (5).

\[
\begin{align*}
  s_j &= i\Omega_j \\
  s_j^* &= -i\Omega_j
\end{align*}
\]  

(5)

Applying the equations (5) to the problem of eigenvalues and eigenvectors, it obtains the equation (6).

\[
K\phi = \Omega^2 M\phi
\]  

(6)

Since \(\lambda_j = -s_j^2 = \Omega_j^2\), with \(j\) varying from 1 to \(n\), it realizes that the eigenvalues \(\lambda_j\) and \(\lambda_j^*\) are real and equal. This allows the solution of this problem to lead directly to \(\Omega_j^2\) and \(\phi_j\), that is, to \(n\) eigenvalues and \(n\) corresponding eigenvectors.

The value of \(\Omega_j^2\) is known as the \(j\)-th eigenvalue or square of the natural frequency and \(\phi_j\) is the corresponding eigenvector, or vibrating mode (EWINS, 2000). The eigenvectors have the characteristic of being orthogonal in relation to the matrix that generate them, being this characteristic of paramount importance for the calculation of frequency response and vibration control problems.

In this work, it is assumed that the damping is very low and can be disregarded. This allows the resonance frequencies to be considered close to natural frequencies (EWINS, 2000). This information is important, because, in the experimental stage, resonance frequencies are measured, not the natural frequencies, since the FRF used to extract the modal parameters is the inertance. That is, the acceleration of the system is measured which when placed in the frequency domain it shows peaks in the resonant frequencies of the system.

If the damping of the system was considerable or it wanted to consider it in the problem, it would be necessary to calculate the damping of each mode.
This would allow the natural frequency of the system to be obtained from the resonance frequency. On the other hand, it could be measured the velocity of the system, because, in the mobility curve, the resonance frequencies will coincide with the natural frequencies of the structure tested.

3.2 FINITE ELEMENT METHOD

For simple problems within the engineering, it is possible to apply the Continuous Mechanics, taking into account that the material is continuous, without voids and ignoring the impact of the crystal structure (RIBEIRO, 2004). However, when it desires to solve very complex problems, it is necessary to use approximate numerical methods, such as the Finite Element Method (FEM).

Before the emergence of this method, the problems were solved from the partial derivative equations that governed the phenomenon, being, commonly, necessary to resort to the Fourier series to obtain the solution (TIMOSHENKO; GOODIER, 1951). However, for complex problems, this alternative was not possible.

Starting from an integral equation that describes the behavior of every complex domain (of volume \( V \)), it is replaced the equation by a sum of integrals in subdomains of simple geometries (of volume \( V_i \)). Since it is possible to calculate all these integrals, the sum of them is done to obtain the solution in the whole domain, in the step called superposition. Each of these subdomains corresponds to a finite element of simple geometry as a line segment, a triangle, a quadrilateral, a tetrahedron, and so on. (AZEVEDO, 2003).

Applying this procedure, it is possible to discretize the entire domain into elements, which results in a mesh of \( n \) nodal points. In these points, the values are equal to that of the exact solution and, the inside the element, the values are a linear combination of nodal solutions by interpolation functions. This approximation allows the obtaining of matrices of mass and rigidity that contains the information of how these quantities are distributed and concentrated in the nodes. From these matrices, it is possible to solve the eigenvalues problem and to determine the natural frequencies and vibration modes of the structure, in the case of modal analysis.
The structure addressed in the work is a simple beam. This component can be mathematically modeled as a combination of the finite elements of Timoshenko and Euler-Bernoulli. The formulation for these elements, as well as the crack considerations, will be presented in the following sections.

3.2.1 Euler-Bernoulli beam model

The Euler-Bernoulli beam model uses the linear theory of elasticity in its formulation and allows obtaining the deflection characteristics of a beam under a load (static or dynamic). However, in order for the model to approximate reality, the beam must meet certain conditions of material, geometry, and displacements. In the material question, the Poisson coefficient must be negligible, have linear behavior, elastic, homogeneous, and with constant density. For geometry, the beam must have a constant cross-section along the length and the cross-section must be symmetrical with respect to the neutral line, which must be contained within the vertical plane.

Finally, in the displacement conditions, it should consider that the angle of rotation is small, that the shear energy and the rotational inertia effects can be neglected, and that the planes perpendicular to the neutral line remain perpendicular to it after deformation.

In other words, the beams that fit this model must have linear and homogeneous material, have the length much bigger than the height and width and have small deformations. Due to these conditions, the cross-sectional area remains perpendicular to the neutral line after deformation, as shown in figure [1].

FIGURE 1 – MODEL OF BEAM THAT RESPECTS THE CONDITIONS FOR BEING OF EULER-BERNOULLI.
From the Newton's laws applied in an infinitesimal element and from the constitutive equations of the material, it obtains the linear partial differential equation that governs the problem. With it, it is possible to obtain, for an element, the mass matrices \( M \) and rigidity \( K \). Considering linear interpolation functions and two degrees of freedom per node (rotation and translation), for the Euler-Bernoulli beam model, the matrices \( M \) and \( K \) are:

\[
[K_e] = \frac{EI}{h_e^2} \begin{bmatrix}
12 & 6h_e & -12 & 6h_e \\
6h_e & 4h_e^2 & -6h_e & 2h_e^2 \\
-12 & -6h_e & 12 & -6h_e \\
6h_e & 2h_e^2 & -6h_e & 4h_e^2 \\
\end{bmatrix}
\] (7)

\[
[M_e] = \frac{\rho Sh_e}{420} \begin{bmatrix}
156 & 22h_e & 54 & -13 \\
22h_e & 4h_e^2 & 13h_e & -3h_e^2 \\
54 & 13h_e & 156 & -22h_e \\
-13 & -3h_e^2 & -22h_e & 4h_e^2 \\
\end{bmatrix}
\] (8)

where \( E \) is the modulus of elasticity, \( \rho \) is the density of the material, \( S \) is the cross-sectional area, \( I \) is the moment of inertia of the second order of area and \( h_e \) is the size of the element. The vector of generalized coordinates and the degrees of freedom of the element are:

\[
d = \{u_1; \ \theta_1; \ u_2; \ \theta_2;\}
\] (9)

where \( u_1 \) and \( u_2 \) are the nodal displacements and \( \theta_1 \) and \( \theta_2 \) the rotations. The figure [2] shows the degrees of freedom for this element.

**FIGURE 2 – DEGREE OF FREEDOM OF THE ELEMENT.**

 SOURCE: the author.

3.2.2 Timoshenko beam model
The Euler-Bernoulli element does not take into account the shear effect. However, this causes the stiffening of the model when under loads that cause shear. In order to avoid this increase in stiffness, Timoshenko's theory considers the effects of shear deformation. For that, it establishes the premise that the cross sections of the beam remain flat, but not, necessarily, perpendicular to the neutral line. Then, in the case of flexion, the model deforms as shown in figure [3], where $\theta$ is rotation of the cross section, $\tau$ the rotation from the shear and $du/dx$ the rotation of the center line of the element. (AZEVEDO, 2003).

**FIGURE 3 – MODEL OF BEAM THAT RESPECTS THE CONDITIONS FOR BEING OF TIMOSHENKO**

Due to these considerations, the rigidity and mass matrices undergo some modifications assuming the form presented in equations (24) and (25)

$$[K_e] = \frac{E}{h_e} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} + \frac{GS}{6h_e} \begin{bmatrix} 6 & 3L & -6 & 3L \\ 3L & 2L^2 & -3L & -3L \\ -6 & -3L & 6 & -3L \\ 3L & L^2 & -3L & 2L^2 \end{bmatrix}$$  \hspace{1cm} (10)

$$[M_e] = \frac{pSh_e}{6} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \frac{ph_e}{6} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$  \hspace{1cm} (11)

where $G$ is the shear modulus of the material and $S$ is the cross-sectional area. The coordinate vector is the same as the Bernoulli element. The second part of these matrices is the shear effect. The development to obtain the matrices for both the Bernoulli element and the Timoshenko element is presented in more detail in a proper bibliography for finite elements. It is suggested to read Azevedo (2003) for more details.
3.2.3 Stiffness of crack

The existence of a crack in a structure causes a local reduction of stiffness and this affects the dynamic characteristics such as modes of vibration and natural frequencies. Due to the variation in stiffness being a function of crack size, the method that relates the variation of flexibility to the length of the failure has been used to determine stable growth in the elastoplastic model of fracture mechanics, known as Unloading Compliance. (IPINÁ, 2004).

In Dimarogonas (1996), there were used the relations of the fracture mechanics to model the crack rigidity in rectangular section beams. In this way, the crack can be modeled from a torsion spring, with stiffness obtained experimentally.

The flexibility is a function of the crack size and it is obtained empirically. For the case of a rectangular section can be expressed by the equation (DIMAROGONAS, 1996).

\[ c = \frac{6\pi h}{bEI} F\left(\frac{a}{h}\right) \]  

(12)

where \( h \) is the height of the beam, \( EI \) the stiffness of the section, \( b \) the width of the beam and \( a \) the size of the crack. For \( 0 < a < h \), the stress intensity factor as a function of the crack size \( F \) is given by:

\[ F\left(\frac{a}{h}\right) = 1,86\left(\frac{a}{h}\right)^2 - 3,95\left(\frac{a}{h}\right)^3 + 16,37\left(\frac{a}{h}\right)^4 + 37,22\left(\frac{a}{h}\right)^5 + 76,81\left(\frac{a}{h}\right)^6 \]
\[ + 126,9\left(\frac{a}{h}\right)^7 + 172,5\left(\frac{a}{h}\right)^8 - 144\left(\frac{a}{h}\right)^9 + 66,6\left(\frac{a}{h}\right)^10 \]  

(13)

Once the flexibility is known, it is possible to calculate the stiffness of the crack as:

\[ K_t = \frac{1}{c} \]  

(14)

3.2.4 MEF in simulation software
The application of MEF in complex structures facilitates problem solving by dividing them into smaller domains of simpler resolution. However, as the mesh is refined, an increasing number of local matrices have to be overlapped. In order to facilitate this, many programs with finite element implemented have been developed, such as ANSYS® and ABAQUS®, for example. In them, in a simplified way, the properties of the material, the mesh, the geometry and the boundary conditions are provided. From this information, the software builds the necessary arrays and solves the problem.

Within ANSYS®, there is the APDL - ANSYS® Parametric Design Language programming interface in which it is possible to model simpler structures, but with a greater control over the properties of the mesh, such as refining and type of element used. This last can be varied according to the problem to be solved. For example, in structures where the thickness is extremely thin, when compared with the other dimensions, elements using the shell formulation such as SHELL181 and SHELL281 may be used in the construction of the mesh. Other elements, such as MASS21 and COMBIN14, can simulate a mass concentrated in one node or a spring with damper connecting two nodes, respectively.

FIGURE 4 – SHELL181 (TO THE LEFT) AND SHELL281 (TO THE RIGHT) ELEMENTS.


FIGURE 5 – MASS21 (TO THE LEFT) AND COMBIN14 (TO THE RIGHT) ELEMENTS.

As seen in Chiquito (2012) and Oliveira (2017), the element PLANE183 can be applied to a beam with a passing crack when it intends to obtain the vibration modes of the same. This element has six to eight nodes and is a 2D element of high order. That is, the interpolation functions of this element are quadratic. Each node has two degrees of freedom, being they, translation in X and in Y.

This type of element can be applied in structures with irregular meshes. (CANONSBURG, 2013). In addition, it can be used in problems that involve a plane state of tensions or deformations, covering cases involving plasticity, hyperelasticity, creep, applied stresses or large deformations and displacements.

![Figure 6 - PLANE183 Element](source: Canonsburg (2013)).

### 3.3 WAVELET TRANSFORM

In signal processing, it is common practice to apply a transform on the signal, taking it from one specific domain to another (OLIVEIRA, 2017). The applicability of this is in the possibility to extract information that it is not evident in the current signal space. In the nineteenth century, Fourier proposed that periodic signals could be decomposed into a sum of sines and cosines. This idea gave rise to the well-known Fourier Transformation (FT), which combines different orthogonal functions to establish a direct relation between signals in the time and frequency domains (BARBOSA, 2001).

In the middle of 80s, based on the concepts of FT, the theoretical foundations for the wavelet transform (WT) were addressed. This transform, as well as Fourier, uses orthogonal functions to map functions of a domain to the so-called "wavelet domain". Within the wavelet system, two types of functions are
used: the ones of scale $\beta$ and the wavelets $\gamma$. The Greek letters $\theta$ and $\psi$ are commonly used for the scale and wavelet functions, respectively. However, these Greek letters are being used in this work to represent autonormalized vibrate modes and eigenvectors. This is why the note has changed.

Within the FT, the sine and cosine functions are local in the frequency domain, but global in that of time, that is, a continuous wave has only one frequency (PRESEZNIAK, 2007). However, the wavelet functions are local in time and frequency domain, because they can be defined in a short interval of time. Due to the differences in the orthogonal functions used in FT and WT, it is noted that the first seeks the content of frequencies contained within the signal while the second searches for details that occur within a short time or frequency interval (OLIVEIRA, 2017).

Since WT has the ability to emphasize the details present within the signal, some works have used it applied on the vibrating modes of the structure. That is, the transform emphasizes the effect of the discontinuity that the crack causes and allow to identify its position. Some examples work that used this approach are Presezniak (2007), Xiang and Liang (2012) Janeliukstis et al. (2017) and Oliveira (2017). Since the vibration mode is being considered a discrete signal, in WT, it is necessary to use a discrete wavelet system in the analysis. These systems include Haar, Symmlet, Coiflet, Daubechies and Cohen-Daubechies-Feauveau. In this work, the systems Daubechies, Symmlet, Coiflet, Meyer, Biortogonal and Biortogonal Reverso will be approached. Thus, the basic properties for defining a wavelet system and the specific characteristics of each system used in this work will be presented. It is worth mentioning that this review is simplified, and, if further understanding is desired, the books Weiss e Wilson (2001) and Misiti et al. (2007) are recommended.

3.3.1 Basic properties of wavelets

Let $\gamma$ be a sufficiently regular and localized function. This function $\gamma \in L^1 \cap L^2$ will be considered a wavelet if it follows the condition of admissibility in the frequency domain:
\[ \int_{\mathbb{R}^+} \frac{|\hat{\gamma}(\omega)|^2}{|\omega|} d\omega = \int_{\mathbb{R}^-} \frac{|\hat{\gamma}(\omega)|^2}{|\omega|} d\omega < +\infty \]  

(15)

where \( \hat{\gamma} \) indicates the FT of \( \gamma \). This condition implies that the integral of the wavelet is zero. This basic requirement is often reinforced by requiring that the function have moments of escape, that is, that equation (15) is respected:

\[ \int_{\mathbb{R}^+} t^k \gamma(t) dt = 0 \quad k = 0, \ldots, m \]  

(16)

In summary, a wavelet will oscillate like a wave and will be located in a region, decreasing until zero, due to damping. Its oscillation is measured by the number of existing moments of escape, and the location is evaluated by the interval where the function is significantly different from zero. Starting from this function \( \gamma \), the wavelets are defined as:

\[ \gamma_{z1z2}(t) = \frac{1}{\sqrt{z1}} \gamma\left(\frac{t - z2}{z1}\right) \]  

(17)

For a function \( f \) of finite energy, define the continuous WF by the function \( C_f \):

\[ C_f(a, b) = \int_{\mathbb{R}} \gamma_{a,b}(t) f(t) dt \]  

(18)

Calculating the function \( C_f \) is equivalent to analyzing the function \( f \) with the wavelet \( \gamma \). The function \( f \) is then described by the coefficients \( C_f(a, b) \) of the wavelet, where \( a \in \mathbb{R}^+ \) and \( b \in \mathbb{R} \). These parameters \( a \) and \( b \) correspond to the scaling factor and the displacement in the time domain, respectively.

In the discrete domain, the scaling and displacement factor are described as \( a = a_0^m \) and \( b = nb_0 \), where \( m \) and \( n \) are integer values and are related to the number of levels to be analyzed for the signal. Applying the parameters in equation (16) and substituting in (17) it obtains:

\[ \gamma_{m,n}(t) = a_0^{-m/2} \gamma\left(\frac{t - nb_0}{a_0^m}\right) \]  

(19)
Using the coefficients (20) and the functions (19), it is possible to reconstruct the signal from equation (21).

\[ f(t) = \sum_n \sum_m C(m, n) \psi_{m,n}(t) \]  

**3.3.2 Daubechies wavelet system**

This family of 1D wavelets was developed by Daubechies and it is the first to allow the manipulation of orthogonal wavelets with compact support (non-periodic) and arbitrary regularity. This system functions as a generalization of the Haar wavelet system by introducing a parameter called the momentum \( M \), which indicates the number of times a function can be derived. Wavelets belonging to this family are given the notation \( dbN \), where \( N \) is the order of the function.

This family contains the Haar wavelet, called within the Daubechies system as \( db1 \), which is the simplest and oldest of the wavelets. It is discontinuous and square, as shown in figure [7].

**FIGURE 7 – SCALING FUNCTION (TO THE LEFT) AND HAAR WAVELET (TO THE RIGHT).**

![Image of Haar wavelet and scaling function]

SOURCE: Misiti et al. (2007).

Mathematically, the Haar wavelet and the scaling function are defined as:

\[ \gamma(x) = \begin{cases} 
0, & x = 0 \\
1, & x \in [0,0.5[ \\
-1, & x \in [0.5,1[ 
\end{cases} \]
\[
\beta(x) = \begin{cases} 
0, & x \notin [0,1] \\
1, & x \in [0,1]
\end{cases}
\quad (23)
\]

With the exception of \(db1\), the wavelets of this family do not have an explicit expression. The figure [8] shows other wavelets belonging to the Daubechies family.

**FIGURE 8 – DAUBECHIES WAVELETS: dbN**

![Daubechies wavelets](image)

SOURCE: Misiti et al. (2007).

Among the properties that these wavelets present, both \(\gamma\) and \(\beta\) support a length of \(2N - 1\), with the number of moments of \(\gamma\) equal to \(N\). Another point is that the \(dbN\) are asymmetric, mainly for low values of \(N\), with the exception of the Haar wavelet. In addition, the regularity of functions increases with the order of it, and the analysis performed is orthogonal.

### 3.3.3 Symlets wavelet system

The symlets (sym\(N\)) constitute a family of almost symmetrical wavelets proposed by Daubechies, having a modified db\(N\) construction. Despite symmetry, the properties of sym\(N\) and db\(N\) are similar. The figure [9] shows the symlets of order from 2 to 8, since sym1 is the Haar wavelet.
3.3.4 Coiflets wavelet system

They were built by Daubechies at the request of Coifman and constitute a family of wavelets with an unusual property. Different from previous families, the wavelet $\gamma$ associated to $\text{coif}N$ has $2N$ null moments, whereas the scaling function $\beta$ has $2N - 1$ null moments and integral equal to 1. Both $\gamma$ and $\beta$ have a support with length of $6N - 1$. The figure [10] shows the coiflets of order 1 to 5.

These wavelets are more symmetrical than the Daubechies. This behavior is also repeated in scale functions. With regard to the length of the support, the $\text{coif}N$ can be compared to $\text{db}3N$ or $\text{sym}3N$. From the point of view of number of zero moments of $\gamma$, they are closer to $\text{db}2N$ and $\text{sym}2N$. 
The main interest in the coiflets lies in the fact that, if \( f \) is a sufficiently regular function, the approximation coefficients \((f, \gamma_{j,k})\) for a sufficiently large \( j \) are well estimated by \( 2^{-j/2} f(2^{-j} k) \), that is, a sample of values of the function \( f \). In addition, if \( f \) is an \( N - 1 \) degree polynomial, the estimate corresponds to the exact value. This property is used to control the difference between the analysis of a given signal by \( \gamma_{j,k} \) and a simplified version of the signal.

3.3.5 Biortogonal wavelet system

The biortogonal wavelets are an extension of orthogonal wavelets. It is well known that symmetry and perfect reconstruction are incompatible when the same finite impulse response filters are used for the decomposition and reconstruction process (MISITI ET AL., 2007). In order to solve this, two wavelets are used in the following way: the first \( \tilde{\gamma} \) is used for the signal analysis and has as coefficients of a function \( f \) equal \( \tilde{c}_{j,k} = \int f(x) \tilde{\gamma}_{j,k}(x) \, dx \). The second \( \gamma \) is used in the synthesis where \( f = \sum_{j,k} \tilde{c}_{j,k} \gamma_{j,k} \).

The wavelets \( \gamma \) and \( \tilde{\gamma} \) are joined by the following duality relation:

\[
\int \tilde{\gamma}_{j,k}(x) \gamma_{j',k'}(x) \, dx = 0 \quad \text{for} \quad k \neq k' \text{ or } j \neq j' \quad (24)
\]

\[
\int \beta_{0,k}(x) \beta_{0,k'}(x) \, dx = 0 \quad \text{for} \quad k \neq k' \quad (25)
\]

This allows concentrating the desired properties for the analysis, such as the number of zero moments, in the wavelet \( \tilde{\gamma} \), whereas the properties of interest for synthesis (regularity, symmetry) can be concentrated in \( \beta \). The figure [11] shows some biortognals (bior).
3.3.6 Meyer's wavelet approximation

The Meyer wavelet ($meyr$) was one of the first proposed wavelets, obtained by Meyer in the mid-1980s. It is an orthogonal function infinitely derivable, but does not have a compact support. The figure [12] shows $\gamma$ and $\beta$.

Both the scaling function and the Meyer wavelet can be described by analytical equations. However, they do not have a compact support, making it difficult to use in practice. This is due to the need to use infinite pulse response filters. In order to correct this, finite pulse response filter approximations of this wavelet were created for fast signal decomposition. The notation for this wavelet is $dme y$. 

source: Misiti et al. (2007).
3.4 TECHNIQUES OF OPTIMIZATION

Optimization is an important tool within the analysis of physical systems (NOCEDAL E WRIGHT, 2006). However, to use it should define a goal to be achieved, some performance parameter of the system studied. This goal can be profit, time spent, energy expended, for example, or it can still be a combination of all of these. Therefore, the objective is related to the characteristics of the system, that is, the variables. Another point to keep in mind, when working with optimization, is the constraints that the problem has. They are, in a simplistic way, the conditions that the solution must respect in order to be considered valid. Examples of constraints would be, the resistance strength of the system material cannot fail, the amount of resources needed not to be exceeded, among others. (ARORA, 2011)

For the problem of the identification of crack in the structure, the variables are the position and depth of the fissure (for the case of the beam). These unknowns are subject to the dimensional constraints of the beam, that is, the position and depth cannot assume negative values nor be greater than the beam length and height, respectively. The objective function, i.e., the value to be minimized, varies depending on the identification method used and will be better addressed in the materials and methods section. In this part of the work, the optimization techniques used will briefly explained.

3.4.1 Genetic algorithm

In 1975, Holland published "Adaptation in Natural and Artificial Systems" being the first approach of Genetic Algorithms. In the 1980s, David E. Goldberg obtained the first successful industrial application of GAs. After that, this technique began to be used to solve problems of optimization and learning of machines. (GABRIEL E DELBEM, 2008)

In biology, it is known that individuals in the same population compete with each other for survival, either through reproduction or the search for resources. Those who are considered more apt will be able to have a greater number of descendants than the less fit ones, passing the characteristics that allowed that evolutionary success ahead. These characteristics that an individual presents are
related to the genes it carries. These genes may assume different values, which imply a variety of expressions for the same characteristic. These values are called alleles and the structures that carry genes are called chromosomes.

Within genetic algorithms, the population is the set of individuals that are being considered as solutions to the problem. In addition, it is also used to create new sets of individuals for analysis. The size of it is directly linked to the programming efficiency and resolution of the problem. When the analyzed population is very small, the solution can converge to a local optimum point, as it can occur the rapid loss of population diversity, reducing the search space. However, if the population considered is very large, there is a loss of computational efficiency, because it will take a much longer time to evaluate all individuals and it will be necessary to work with more computational resources. About the individuals, as it said, each one is a solution of the problem and one or more chromosomes form it, which depends of the technique used or the problem to be solved. These chromosomes contain the parameters that allow the individual to be a solution, the genes. They are formed by the alleles that can assume different values in the solution.

Different approaches can be used to code of the parameters in the genes. The main one is to represent each attribute as a sequence of bits and concatenate them to assemble the chromosome for the individual. Another form of coding may be to use binary language, as shown in Holland 1975 and Caruana 1988.

Through a certain function, the individuals are evaluated and, depending on the value obtained in the calculation, one has the aptitude of it. This function is of paramount importance within genetic algorithms, because it will determine when a solution is good or not. In addition, it will determine which individuals are most apt to pass on the characteristics and which are not. If the chosen function is not able to separate good solutions from bad, optimal solutions can be eliminated and the program will explore less promising solution. This will increase the resolution time.

Inside the genetic algorithm, there are different steps to obtain the resolution. One of them is the selection step that occur in two different moments. These are to make the choice of individuals "parents" and to choose the individuals on which the genetic operators will be applied.
In the selection of "parents" individuals, it will try to be simulated the biological process of selection, in which the more capable parents tend to generate more descendants than the less able. In other words, the best sets of solutions will generate more individuals than the worst. In addition, even the groups that did not present good performance will still have descendants, due they may contain favorable characteristics in the search for the optimal solution to the problem.

In the case of the selection of the individuals in which the operators will be applied, it can be used several approaches and the main are method of Roulette and of Tournament. The second one has an advantage over the first one, in the matter of computational time, because it does not use the whole population in the evaluation. It is worth noting that these techniques are not the only ones used, and they may vary for each type of problem or approach desired.

The operator of crossover (or recombination) can be considered the dominant genetic operator. It starts from the premise of generating new individuals by mixing the characteristics of "parent" solutions, switching or matching genes. From this, it is possible to obtain potentially better offspring, as they carry the characteristics of both matrix solutions. Within GAs, it is the main mechanism of reproduction (Goldberg 1989). Amount of information that will be exchanged or combined is called the crossover rate.

FIGURE 13 – CROSSOVER (OR RECOMBINATION) OPERATOR.

SOURCE: Gabriel; Delbem (2008)

In the mutation operation, one or more genes are randomly modified within a chromosome. This variation is important, because it allows inserting new characteristics in the population that previously did not exist or appeared in few
individuals. This operator is necessary to maintain the genetic diversity within the population, besides allowing being possible to reach any point of the search space. However, the mutation rate, the probability of these changes occurring, is generally small, since the new individual may be potentially worse than the original.

**FIGURE 14 – MUTATION OPERATOR.**

![Mutation Operator Diagram]

**SOURCE:** Gabriel; Delbem (2008)

Regarding the applicability of GAs, in order to apply this methodology to an optimization problem, it must be possible to represent the solutions in the form of a genetic code. In addition, there must be ways to assess the quality of the individual, as well as a criterion that can determine which solution may remain and which should be withdrawn.

About the genetic operators, they must be previously defined, in a way that can combine individuals to create new solutions, as well as the insertion of new characteristics in the population, guaranteeing the diversity and possibility of generating innovative solutions. Finally, the initial population must be of sufficient size and diversity that it is not led directly to a local minimum, since small and very similar groups end up stagnating the search process.

### 3.4.2 Sequential Quadratic Programming (SQP)

Unlike the GA, which applies statistics, the SQP method is a nonlinear optimization technique that, from an initial guess, executes an iterative process in search of the solution. This method is one of the most efficient non-linear for constrained optimization and can be applied to small or large scale problems (NOCEDAL, 2006). It is considered the applicability of the SQP methodology for nonlinear optimization of problems with the following formulation (HOPPE, 2006):
\[
\begin{align*}
\text{minimize } & \quad f(x) \\
\text{with } & \quad x \in \mathbb{R}^n \\
\text{subject to } & \quad h(x) = 0 \\
& \quad g(x) \leq 0,
\end{align*}
\]

where \( f: \mathbb{R}^n \rightarrow \mathbb{R} \) is the objective function, while \( h: \mathbb{R}^n \rightarrow \mathbb{R}^m \) and \( g: \mathbb{R}^n \rightarrow \mathbb{R}^p \) describe the constraints of equality and inequality. For an iteration point \( x_k \), with \( k \in \mathbb{N}_0 \), Quadratic Programming (QP) technique is applied for solving the subproblem. After that, the solution obtained to construct the new iteration step \( x_{k+1} \) is used. The process will be repeated so that the sequence \( (x_k) \) converges to a local minimum \( x^* \). This method is similar to the Newton and Quasi-Newton Method, however, the presence of constraints makes the implementation of the SQP method a bit more complicated (Hoppe, 2006). Unlike GA, the mathematical formulation behind this method is more complicated to explain briefly. Therefore, if it is of interest to obtain a better explanation about SQP and QP, it is suggested to consult Hoppe (2006) and Nocedal (2006).

3.4.3 Nelder-Mead Method

This method was introduced in 1965 and has become one of the most widely used methods in unrestricted problems, being especially popular in the fields of chemistry, chemical engineering and medicine (Lagarias, 1998 e Nocedal, 2006). In this method, it is sought to minimize the real value of a scalar, obtained by a nonlinear function of \( n \) real variables, using only the values obtained from the function. That is, it does not use any derivative information, either implicitly or explicitly, characterizing this method as a Derivative-Free Optimization (DFO). Like other direct search methods, in each iterative step a non-degenerate \( n \)-dimensional geometric figure with non-zero volume is created. This figure is composed of a convex hull of \( n + 1 \) vertices and they are associated with the values of the function.

In order to construct an algorithm for this method, it is necessary to define some important parameters such as reflection (\( \mu \)), expansion (\( \chi \)), contraction (\( \kappa \)) and shrinkage (\( \theta \)) coefficients. According to Nelder e Mead (1965), these parameters should satisfy relations:
\[ \mu > 0, \quad \chi > 1, \quad \chi > \mu, \quad 0 < \kappa < 1, \quad e \quad 0 < \vartheta < 1 \quad (26) \]

The equation \( \chi > \mu \) was not directly referenced, but can be deduced by the description of the function of the algorithm, and, generally, these parameters assume the following values (NOCEDAL, 2006):

\[ \mu = 1, \quad \chi = 2, \quad \kappa = \frac{1}{2}, \quad e \quad \vartheta = \frac{1}{2} \quad (27) \]

In each iteration \( k \) (sendo \( k \geq 0 \)), a nondegenerate simplex \( \Delta_k \) whose \( n + 1 \) vertices are points whose coordinates are within \( \mathbb{R}^n \). They are organized in such a way that:

\[ f_1(k) \leq f_2(k) \leq \ldots \leq f_{n+1}(k) \quad (28) \]

where \( f_i(k) \) is the function evaluated at point \( x_i(k) \). The objective is the minimization, so, \( x_{n+1}(k) \) is the worst point, \( x_1(k) \) the best and \( x_n(k) \) the next worse. In generation \( k + 1 \), this set of vertices is changed, so that the simplex \( \Delta_{k+1} \) is different from \( \Delta_k \). The article by Nelder and Mead opens space for numerous interpretations about the inequalities and rules of stopping. Thus, the algorithm used by the code in MATLAB will be presented.

Initially, the points are organized in order to satisfy the equation (28). After this step, the reflection point \( x_r \) is calculated as follows:

\[ x_r = \bar{x} + \mu (\bar{x} - x_{n+1}) = (1 + \mu) \bar{x} - \mu x_{n+1} \quad (29) \]

where \( \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \) is the centroid of the \( n \) best points. That is, all vertices minus \( x_{n+1} \). After the calculation of the point \( x_r \), it is performed the evaluation of the function in it \( (f_r) \). If it is greater than or equal to the best evaluation and less than the second worst, that is, if the relation \( f_1 \leq f_r < f_n \) is satisfied, the reflection is accepted and the iteration is closed. If this condition is not respected, it may be necessary to apply the expansion or contraction parameters, depending on the
value of $f_r$. If it is less than $f_1$, the expansion step is performed by calculating a point $x_e$ as follows:

$$x_e = \bar{x} + \chi (x_r - \bar{x}) = \bar{x} + \mu \chi \bar{x} - x_{n+1} = (1 + \mu \chi)\bar{x} - \mu \chi x_{n+1} \quad (30)$$

It evaluates $f_e$ as $f(x_e)$ and checks whether it is less than $f_r$. If it is, it accepts $x_e$ and ends the iteration, otherwise it accepts $x_r$. However, if $f_n \leq f_r$, it is applied contraction step between $\bar{x}$ and the best between $x_{n+1}$ and $x_r$. This step can be an internal or external contraction. The latter occurs if $f_n \leq f_r < f_{n+1}$ and a point $x_c$ is calculated:

$$x_c = \bar{x} + \kappa (x_r - \bar{x}) = \bar{x} + \mu \kappa \bar{x} - x_{n+1} = (1 + \mu \kappa)\bar{x} - \mu \kappa x_{n+1} \quad (31)$$

It evaluates $f_c$ and compare with $f_r$. If $f_c \leq f_r$, the point $x_c$ is accepted and ends the iteration. Otherwise the shrinkage step applies.

The internal contraction occurs when $f_{n+1} \leq f_r$, being necessary the calculation of $x_{cc}$:

$$x_{cc} = \bar{x} - \kappa (\bar{x} - x_{n+1}) = (1 + \kappa)\bar{x} + \gamma x_{n+1} \quad (32)$$

It evaluates $f_{cc}$ and compare with $f_{n+1}$. If $f_{cc} < f_{n+1}$, $x_{cc}$ is accepted and ends the iteration. Otherwise the shrinkage step applies.

In the shrinking step, all vertices, except the first, are substituted. The new coordinates are calculated by:

$$v_i = x_1 + \vartheta (x_i - x_1) \quad (33)$$

where $i$ ranges from 2 to $n + 1$. The operations of reflection, expansion, internal contraction, external contraction and shrinkage are presented in figures [15], [16], [17], [18] and [19].
FIGURE 15 – NELDER-MEAD SIMPLEX AFTER REFLECTION STEP

 SOURCE: Lagarias et al. (1998)

FIGURE 16 – NELDER-MEAD SIMPLEX AFTER EXPANSION STEP

 SOURCE: Lagarias et al. (1998)

FIGURE 17 – NELDER-MEAD SIMPLEX AFTER EXTERNAL CONTRACTION

 SOURCE: Lagarias et al. (1998)

FIGURE 18 – NELDER-MEAD SIMPLEX AFTER INTERNAL CONTRACTION

 SOURCE: Lagarias et al. (1998)
FIGURE 19 – NELDER-MEAD SIMPLEX AFTER SHRINKAGE STEP

SOURCE: Lagarias et al. (1998)
4 MATERIALS AND METHODS

As shown in the review, different methodologies for identification of crack parameters can be used. However, even though the methods are different, they all follow the same operating procedure. It is initially defined which methodology will be used in the identification. This may be through some graphical method, monitoring of some modal parameter, among others.

After that, the numerical model is chosen for the structure and for the crack, as well as the optimization technique that will be used to obtain the crack parameters. It is worth mentioning that, depending on the applied identification method, both the crack and beam modeling step and the chosen optimization technique can be omitted. This is due to the fact that some methods do not need them, the experimental data being sufficient information for identification.

Finally, the technique of acquiring the necessary vibrational characteristics is chosen. This acquisition can occur by experimental modal analysis (EMA) or by operational modal analysis (OMA). The reason for this step to be put last is that, depending on the method of identification chosen, the experimental procedure may be different. That is, the input of experimental data is not the same for all methodologies, thus varying what will be measured and how it will be measured.

In the figure [20], a flowchart is presented with the resolution of the identification problem of the crack parameters. It shows the possible choices for each step.
In the following sections will be explained the methods of identification that will be addressed at work, the numerical models and techniques of optimization applied, as well as the data acquisition procedures used.

4.1 METHOD OF IDENTIFICATION

It consists of the methodology applied to identify the parameters of position and depth of the crack. Depending on the method explained the numerical-computational model and optimization technique can be omitted. In this dissertation will be approached five different methods for the identification of the crack, each having its peculiarities.

4.1.1 Wavelet Method

In this methodology the Wavelet Transform is applied on the vibration modes obtained from the structure, as proposed in the works of Chasalevris and Papadopoulos (2006), Presezniak (2007), Zhang et al. (2009), Xiang and Liang (2012), Masoumi and Ashory (2014) and Janeliukstis et al. (2017).

As explained in the theoretical fundamentation, when WT is applied on a given curve, the details become more evident as the level being analyzed rises.
However, for multi-level analysis, it is necessary to interpolate mode data, because the number of points decreases from a lower level to a higher level of the filter.

Initially, the vibration modes of the structure are obtained and a spline interpolation step is performed, in order to obtain a greater number of points than was actually measured. After that, it is selected the wavelet that will used and in how many levels the signal will be analyzed, obtaining for each of them a new curve. In order to identify the position, the existence of peaks or valleys in these curve is verified, being that depending on the wavelet the location will be determined by the maximum or the minimum. This information will become clearer in the results section.

In order to make the work more comprehensive, eight wavelets were used, being: \textit{db1} and \textit{db2} (from the Dauchebies family), \textit{coif1} (from the Coiflets system), \textit{sym2} and \textit{sym4} (from the Symlets group), \textit{dme1} (Meyer's discrete), \textit{bi1} or 1.5 and \textit{rbio1.1} (a biortogonal and another biotogonal reverse). The purpose is to verify which one performs best, in which the position is identified by the maximum and in which by the minimum. Regarding the level, the decompositions were evaluated up to the 10th level, making it possible to identify small cracks. As the level used is relatively high, the data was interpolated using 3800, 7600, 11400, or 15200 points. The variation was aimed at verifying how the identification varies as the number of points increases.

The modes were analyzed separately, and the fissure was identified for each without combining the information with the others. This allows you to check which modes allow a better characterization of the crack, and it is not a simple task to combine the decompositions of different modes without corrupting the results.

Different from the previous works, the numerical derivative of the curve obtained in each level and by each wavelet was also calculated. The reason for this is in the fact that it was realized that where there was crack, the curve obtained by WT showed a sudden change in value, in some cases. This variation could be used as an identifier, and it is necessary only to derive the functions numerically. It is worth noting that this abrupt variation occurred only when the curve did not present peaks or valleys exactly in the position of the crack, and the use of the derivative was an alternative form of identification.
The advantage of this method over the others is that it is not necessary to define any numerical computational model or optimization technique. This allows a resolution of the inverse problem in a much shorter time and with fewer calculations than the others. The figure [21] shows the options available within this methodology.

4.1.2 Combined Method

In this method are applied the concepts of localized flexibilization approached by Dresig and Holzweißig (2006) in the machine dynamics book. The basic idea behind this method is that frequency changes due to damage are usually small and do not indicate the location of the damage in a clear or simple manner (DRESIG and HOLZWEIßIG, 2006).

The proposed method is based on an approach that combines the natural frequencies and modes of vibration. The equation (34) describes the natural frequency sensitivity:

\[
\Delta \omega_i^2 = \frac{\phi_{i0}^T \Delta K \phi_{i0}}{\phi_{i0}^T M_0 \phi_{i0}} - \omega_{i0}^2 \frac{\phi_{i0}^T \Delta M \phi_{i0}}{\phi_{i0}^T M_0 \phi_{i0}}
\]  

(34)
where subscript 0 indicates the parameters of the intact structure and \( i \) indicates the vibrate mode being analyzed. That is, \( \phi_{i0} \) and \( \omega_{i0} \) are the \( i \)-th vibration mode and the \( i \)-th natural frequency of the intact structure. Now \( \Delta K, \Delta M \) and \( M_0 \) correspond to the variation of the stiffness matrix, the mass matrix and the mass matrix of the intact structure, respectively. Since crack causes a variation in rigidity, the mass variation matrix can be disregarded. In order for this equation to be applied it is necessary to measure the natural frequencies (\( \omega \)) and the vibration modes (\( \phi \)).

It is assumed that the crack occurs between two neighboring DOFs. It causes a loss of local stiffness of the structure, this variation being described by equation (35).

\[
\Delta K = \left[ \begin{array}{cc} \Delta k & -\Delta k \\ -\Delta k & \Delta k \end{array} \right]
\]  

where \( \Delta k \) is slightly negative because there is a flexibilization of the region. This parameter allows to obtain the magnitude of the fissure, being necessary to define how it is related to the depth of the crack. This relation will be tied to the numerical-computational model that one wishes to use. In equations [39] and [40] the values of \( \Delta K \) for element of Euler-Bernoulli and of Timoshenko, respectively, are presented. In this dissertation the first one will be used, since the numerical model constructed for MATLAB crack uses this type of element in conjunction with the torsional spring, not Timoshenko's. Thus, the method using the Timoshenko formulation was presented here only with an illustrative character.

\[
\Delta K = -\frac{E b (h^3 - (h - a)h)}{h_e^3} \left[ \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right]
\]  \hspace{1cm} (36)

\[
\Delta K = -\frac{E b (h - (h - a))}{3h_e} \left[ \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right]
\]  \hspace{1cm} (37)

where \( h_e \) is the length of the element, \( E \) is Young's modulus of elasticity, \( b \) is the cross-sectional base, \( h \) is the height and \( a \) is the depth of the crack. In both equations, the negative signal is put forward because it is the loss of rigidity, so
that the $\Delta k$ are negative. A point that must be highlighted is that in the operational or experimental modal analysis only the nodal displacements in the vibration modes are measured. Therefore, it is not possible to tell how the nodes' rotations are changing with the crack growth. This makes that in the matrix $\Delta K$ there are only the values referring to pure translation, the mixed or rotational components of the nodes in the elements of Euler-Bernoulli and Timoshenko being disregarded. This will have an impact on the accuracy of the identification and will be better explained in the results stage.

Manipulating equation (34), it has the following relations:

$$\omega_{iN} = \sqrt{(\omega_{i0}^2 + \phi_{i0}^T \Delta K_{mod} \phi_{i0})}$$

(38)

$$r = \sum_{i: todos os modos} \left(\frac{\omega_{iN} - \omega_{iE}}{\omega_{iE}}\right)^2$$

(39)

where $\omega_{iN}$ is the $i$-th natural frequency obtained numerically, $\omega_{iE}$ the $i$-th natural frequency obtained experimentally, $\omega_{i0}$ the $i$-th natural frequency of the intact structure obtained experimentally, $\phi_{i0}$ the $i$-th way of vibrating of the intact structure and $\Delta K_{mod}$ the variation of the modified stiffness matrix. The latter is a result of the division of $\Delta K$ by $\phi_{i0}^T M_0 \phi_{i0}$, which is the parameter of the $i$-th modal mass. If the results are obtained by EMA, it is possible to obtain this scale factor for the modes and to estimate the severity of the damage correctly. However, when the vibrational data are obtained by OMA, the values of modal masses may not be available, making it impossible to calculate the depth of the crack.

When it uses the equation (39) as objective function to be minimized, it is possible to find the position and depth of the crack. However, as the error function only works with frequencies, the location of the fissure in symmetric structures can be compromised. That is, the crack on the right side results in the same frequency variation as a crack on the left side, by the symmetry of the structure.

To obtain the minimum error, it is necessary to define an optimization technique to be used. As mentioned previously, three different techniques going to be used, being them only GA, only NLOT and a hybrid between GA and NLOT.
The operation of each one going to be better explained in the Optimization Techniques section of this chapter.

As can be seen, the step of the greatest computational expense of the identification by the combined method is the optimization. This time is measured for each crack in order to be possible to compare them. In addition, this method does not present many possibilities of variation, being only possible to choose the data acquisition technique and the optimization tool. The figure [22] shows the parameters for the combined method.

**FIGURA 22 – FLOWCHART WITH POSSIBLE PARAMETERS OF THE COMBINED METHOD**

SOURCE: the author.

4.1.3 Graphical Method

As was done by Hu and Liang (1993), Patil and Maiti (2003), Xiang et al. (2006), Chasalevris and Papadopoulos (2006), Xiang et al. (2007) and Zhang et al. (2009), the position and depth identification is obtained by overlapping the frequency variation curves for each mode.

The method consists in firstly obtaining the natural frequency variations of the structure cracked through an experiment step. After that, a numerical-computational model will be used to describe the vibrational behavior of the beam. This model is calibrated for the case of null depth cracking, avoiding that the numerical and experimental frequencies of the intact structure are different.
Once calibration is done, the crack parameters are varied in order to construct a frequency variation surface for each vibrating mode. These maps are sectioned according to the differences between natural frequencies measured experimentally, obtaining curves for each mode. These curves are overlaid and where they intersect will match the location and depth of the fissure.

For this method to work, at least two variations of natural frequencies are required, since different pairs of position and depth can generate the same variation in a certain mode. However, only one pair can generate one variation X in one mode and a variation Y in another. This corroborates with what it was explained by Rizos et al. (1990), Narkis (1994) and other researchers.

In order to determine the best combination of modes, errors in crack identification for all possible arrangements of two to four eigenvectors were calculated. The table [1] shows the arrangements of the mode curves used.

<table>
<thead>
<tr>
<th>ARRANGEMENT</th>
<th>MODES USED</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1st AND 2nd</td>
</tr>
<tr>
<td>2</td>
<td>1st AND 3rd</td>
</tr>
<tr>
<td>3</td>
<td>1st AND 4th</td>
</tr>
<tr>
<td>4</td>
<td>2nd AND 3rd</td>
</tr>
<tr>
<td>5</td>
<td>2nd AND 4th</td>
</tr>
<tr>
<td>6</td>
<td>3rd AND 4th</td>
</tr>
<tr>
<td>7</td>
<td>1st, 2nd AND 3rd</td>
</tr>
<tr>
<td>8</td>
<td>1st, 2nd AND 4th</td>
</tr>
<tr>
<td>9</td>
<td>1st, 3rd AND 4th</td>
</tr>
<tr>
<td>10</td>
<td>2nd, 3rd AND 4th</td>
</tr>
<tr>
<td>11</td>
<td>1st, 2nd, 3rd AND 4th</td>
</tr>
</tbody>
</table>

SOURCE: the author.

The negative point of this technique is the high computational time spent only to generate the maps for later overlap. The time increases with the mesh refining of the surfaces. However, the identification itself occurs very quickly, in addition to the fact that the process for creating surfaces only needs to be done once. That is, maps are generated and different cracks are identified by superimposing different sections of the same graphs.
There is no need for an optimization technique, since the solution is obtained graphically. However, for the creation of the surfaces it is necessary a numerical model whose mesh and complexity can influence the computational time spent. The figure [23] shows the possible parameters for the graphical method.

**FIGURA 23 – FLOWCHART WITH THE POSSIBLE PARAMETERS OF THE GRAPHIC METHOD**

![Flowchart](image)

**SOURCE:** the author.

### 4.1.4 Frequency Method

This methodology is an improvement of the graphical method because, instead of calculating the frequencies at all points, an optimization tool is applied, in order to find the pair of location and depth that generate the desired pattern of variations in the frequencies of the structure. This method was applied in the works of Rizos et al. (1990), Xiang et al. (2008), Behzad et al. (2013) and Oliveira (2017).

This method can use the natural frequencies directly, comparing the experimental with the numerical, or the frequency differences, comparing the variations between intact and cracked experimental with the variations between intact and cracked numerical. The application of both variant will be approached in this work, since the precision and computational expense can change.
In equations (40) and (41), the vectors are presented when working with the frequencies directly and when the differences between frequencies are used, respectively.

\[ g_i = \left| \frac{\omega_{iN} - \omega_{iE}}{\omega_{iE0}} \right| \] (40)

\[ g_i = \left| \frac{\omega_{iN} - \omega_{iN0}}{\omega_{iN0}} \right| - \left| \frac{\omega_{iE} - \omega_{iE0}}{\omega_{iE0}} \right| \] (41)

where \( \omega_{iN} \) and \( \omega_{iN0} \) are the ith natural frequencies of the cracked and intact numerical structure, respectively, and \( \omega_{iE} \) and \( \omega_{iE0} \) are the ith natural frequencies of the cracked and intact experimental structure, respectively. In both equations, relative values were used, i.e., divided by the intact (numerical or experimental) frequencies, because, as the number of the mode used increases, the greater the frequency variations. The goal is to prevent higher order modes from having greater influence on the solution. The vector \( g \) is used to construct the necessary objective function. It is presented in equation (42).

\[ r = \log(e^{-40} + \| g \|) \] (42)

Since the variations are subtle, the logarithm function on the norm of the difference vector \( g \) was applied. The presence of \( e^{-40} \) serves to establish a limit value for \( r \), avoiding that the logarithm is applied to a null vector of differences. The possibility of this happening is low, but the precaution is necessary.

Unlike the graphical, combined and wavelet method, the frequency method needs both the model and the optimization technique. Both will influence accuracy and computational time. The figure [24] shows the possible choices for the frequency method parameters.
4.1.5 Vibrate Mode Method (or Mode Method)

The vibration modes of a structure are more sensitive to its integrity than the natural frequencies (OLIVEIRA ET AL., 2017). This statement raised the possibility of using only this modal parameter in the identification, but without the application of the wavelet transform. This application was little studied, and the work of Hou and Lu (2016) is one of the few to perform the identification of cracks through the mode. However, it is noteworthy that, in their work, the mode was applied in conjunction with the natural frequencies.

One of the reasons for the lack of applicability of the vibration mode is the difficulty of measuring it reliably, since measurement noise hinders the acquisition (CAO ET AL., 2014). In addition, a good number of points are required that the chosen mode can be rebuilt.

In this dissertation, it will try to identify cracks through the vibration modes acquired at 19 points along the beam. In addition, this method presents variants that make it interesting. That is, there is the possibility of working with the modes by interpolating them or not, as well as using them directly or the difference between them. The variants are presented in table [2]. It is worth noting that this
interpolation was only 500 points, and it was not studied whether or not the number of points influenced the identification.

<table>
<thead>
<tr>
<th>VARIANT</th>
<th>INTERPOLATION</th>
<th>RELATION BETWEEN MODES</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>YES</td>
<td>DIFFERENCE</td>
</tr>
<tr>
<td>5.2</td>
<td>YES</td>
<td>ABSOLUTE</td>
</tr>
<tr>
<td>5.3</td>
<td>NO</td>
<td>DIFFERENCE</td>
</tr>
<tr>
<td>5.4</td>
<td>NO</td>
<td>ABSOLUTE</td>
</tr>
</tbody>
</table>

Like the frequency method, in the variant of the methodology using the direct mode, the numerical and experimental results are compared and, for each natural frequency, the difference vector normalized is calculated. In the variant of the method that uses the differences of the modes, it initially calculates the norms of the numerical and experimental vectors of the variation of the modes of the structure with and without crack. After that, the component of the error vector for that natural frequency is given by the absolute value of the difference between the norms previously calculated. The equations used in these two possibilities are presented in equations (43) and (44).

\[ g_i = \| \phi_{iN} - \phi_{iE} \| \]  
\[ g_i = | \| \phi_{iN} - \phi_{iN0} \| - \| \phi_{iE} - \phi_{iE0} \| | \]  

where \( \phi_{iN} \) and \( \phi_{iN0} \) are the \( i \)-th numerical vibration modes of the cracked and intact structure, respectively, and \( \phi_{iE} \) and \( \phi_{iE0} \) are the \( i \)-th experimental vibration modes of the cracked and intact structure, respectively.

The problem with these operations involving modes is the fact that everyone must be on the same scale factor. That is, all must be previously normalized before being used in the calculation. In addition, the orientation between the modes compared should be the same. If one of them starts with a positive value, the other must also start with a positive value. The reason for this is to avoid that the difference between them is erroneously calculated, because, if the orientation is reversed, the variation in each coordinate can be twice the displacement of that node.
Having these error vectors \( g \), the objective function can be obtained in a manner similar to the frequency method. That is, by equation (45).

\[
    r = \log(e^{-40} + \| g \|) \tag{45}
\]

The same explanations concerning the logarithm function and the presence of the value of \( e^{-40} \) in the equation given in the frequency method serve in the mode method. In addition, the numerical models and optimization techniques are the same as those previously used. The only difference is in the hybrid optimization technique that, instead of using Nelder-Mead as NLOT, uses SQP. The reason for this lies in the fact that for the mode method, the absence of boundary conditions led, eventually, to locations and depths inconsistent with the beam dimensions. The figure [25] shows a flow chart with the possible choices for the parameters of the method of the mode.

**FIGURE 25 – FLOWCHART WITH POSSIBLE PARAMETERS FOR THE MODE METHOD.**

![Flowchart](image)

SOURCE: the author.

### 4.2 NUMERICAL MODEL

As regards numerical models used to describe the behavior of both crack and intact structures, two different models were used. In this section will be presented each as well as given explanations on what jobs they are referenced.
4.2.1 MATLAB Model

The first model was built inside MATLAB and used a methodology similar to that proposed by Presezniak (2007). In his work, the intact part was modeled by beam elements with Timoshenko's formulation, except the elements adjacent to the crack, because in these the adopted formulation was Euler-Bernoulli's formulation. The fissure was modeled as a torsional spring, as was done in several other works.

For this dissertation, some modifications will be proposed for the model, in order to facilitate the programming as well as to allow a better description of the behavior of the structure. Thus, the Euler-Bernoulli's formulation will not be used for the elements adjacent to the fissure, but that of Timoshenko, as was proposed for the intact region of the structure. For the crack, the torsional spring will be applied in parallel with a reduced-section Euler-Bernoulli element. That is, its height will equal that of the beam minus the depth of the crack. The figures [26] and [27] show the model used by Presezniax and the one proposed for this dissertation.

Since $L$ is the length of the beam and $h$ is the height. The darker elements were constructed using the Euler-Bernoulli formulation, while the clearer ones were made with Tymoshenko's formulation. The stiffness of the crack is shaped...
by the crack, and, together with the darker element, they represent the effect of the failure on the structure.

A model similar that could be studied uses Euler elements for the beam and Timoshenko elements for the fissure. Some papers that give theoretical support to this model are those of works by Qian et al. (1990), Friswell and Penny (2002) and Cacciola et al. (2003), for example. However, such model was not implemented because of the time required to test and analyze the results. It is cited here only as a suggestion for future works.

4.2.2 ANSYS Model

The second model was built within the APDL - ANSYS® Parametric Design Language. In it was applied the element PLANE183, with the beam being modeled as an area of dimensions equal to the length and height of the structure. The crack was considered as a void in this area, and it has a rectangular shape and a triangular tip. This model was presented in the works of Chiquito (2012) and Oliveira (2017). However, different of proposed by Chiquito, it was not used an element with a singularity point (quarter-point), since its addition does not contribute to the calculation of natural modes and frequencies, as discussed by Oliveira (2017).

In the figures [28], [29] and [30], the schematic drawing of the second model is presented and model applied inside the ANSYS. It is worth mentioning that the schematic illustration is out of scale and is intended only to show how the geometry was built into the software, especially the crack.
Models involving continuous mechanics and explicit equations for cracked structures were not studied, due to the difficulty of modeling. In addition, the works that used continuum mechanics models only looked for frequencies and not the ways of vibrating. These models is not accord of the proposal of this dissertation, to verify the applicability also of the mode in the identification of the crack.

The choice of the model implemented in ANSYS® or MATLAB® will influence in the identification, giving different computational time and accuracy. This difference will be analyzed within the results analysis section.

4.3 TECHNIQUES OF OPTIMIZATION

In some methodologies, in order to identify the crack parameters, it is necessary to solve the inverse problem, which can be summarized as obtaining the least error of an objective function. That is, it is necessary to minimize a function according to some boundary conditions.

Thus, different techniques can be proposed to solve this problem, and, in this section, the tools applied in this work are presented. It will also show the parameters used, as well as the procedure adopted.
4.3.1 Nonlinear Optimization Technique

One of the proposed tools uses exclusively nonlinear optimization technique, and the chosen method is the Sequential Programing Quadratic (SPQ). This is because the problem has the constraints that the crack cannot be located at a position greater than the length of the beam or with a depth greater than the available height. In addition, for the both parameters, the negative values are not allowed.

The problem of using only this technique lies in the fact that it is a tool for obtaining local and non-global minima. In order to get around this issue, the problem is solved starting from different initial guesses. The dispersion of these guesses within the domain is presented in the figure [31].

**FIGURE 31 – NORMALIZED COORDINATES OF THE INITIAL GUESSES.**

The reason for not being used for the normalized position no value greater than 0.5 is that the structure is symmetric, i.e., a crack on the right hand side has the same effect as a crack on the left side. For depth, no kick was taken at 0.25, to make it easier to find small cracks. That is, to allow for low depths the vicinity around zero to be favored.

After obtaining these local minimums, through the solution of the problem by SPQ, the obtained values are combined in order to provide a new quantity of
points. This combination uses the normalized position of one solution and the normalized depth of another.

In all of these new points, the error function will be evaluated. This step aims to look for the global minimum in a more reliable way by making a comparison between the solutions found from different initial kicks. In these comparisons, the combinations are included too, because they can lead to minor errors.

The pair of position and depth that has the lowest value of the objective function will be considered the global minimum and, on it, the precision calculations will be performed. The table [3] shows the coordinates of the points obtained by the combination.

### TABLE 3 – COMBINATION OF THE COORDINATES.

<table>
<thead>
<tr>
<th>xt/L / a/h</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x1,a1)</td>
<td>(x1,a2)</td>
<td>(x1,a3)</td>
<td>(x1,a4)</td>
<td>(x1,a5)</td>
<td>(x1,a6)</td>
</tr>
<tr>
<td>2</td>
<td>(x2,a1)</td>
<td>(x2,a2)</td>
<td>(x2,a3)</td>
<td>(x2,a4)</td>
<td>(x2,a5)</td>
<td>(x2,a6)</td>
</tr>
<tr>
<td>3</td>
<td>(x3,a1)</td>
<td>(x3,a2)</td>
<td>(x3,a3)</td>
<td>(x3,a4)</td>
<td>(x3,a5)</td>
<td>(x3,a6)</td>
</tr>
<tr>
<td>4</td>
<td>(x4,a1)</td>
<td>(x4,a2)</td>
<td>(x4,a3)</td>
<td>(x4,a4)</td>
<td>(x4,a5)</td>
<td>(x4,a6)</td>
</tr>
<tr>
<td>5</td>
<td>(x5,a1)</td>
<td>(x5,a2)</td>
<td>(x5,a3)</td>
<td>(x5,a4)</td>
<td>(x5,a5)</td>
<td>(x5,a6)</td>
</tr>
<tr>
<td>6</td>
<td>(x6,a1)</td>
<td>(x6,a2)</td>
<td>(x6,a3)</td>
<td>(x6,a4)</td>
<td>(x6,a5)</td>
<td>(x6,a6)</td>
</tr>
</tbody>
</table>

### 4.3.2 Genetic Algorithm

Unlike the SQP, the genetic algorithm is a search method of global minimum. As the objective function is not smooth with well-defined minimums, high values are considered for both crossover and mutation rates. In addition, at the recombination, it is used the 'crossoverintermediate' technique and, at the mutation, it is applied the 'mutationuniform' method. These tools are GA options and already are implemented in MATLAB.

The crossover tool consists, according to a geometric explanation, in creating "daughter" solutions within the line that connects the coordinates of the "parent" solutions. The mutation technique consists in to select a fraction of the individuals' inputs, to calculate the probability of them mutating, and to replace them with a randomly selected value. This number is uniformly selected in the input range.
Other GA tools that have been applied to the problem are the presence of elitism and migration. The first consists in to choose a fraction of the best individuals from the previous generation and keep them in the next generation. Migration refers to the possibility that the worst solutions of one subpopulation can be replaced by the best solutions of another.

One point that should be highlighted is that the computational cost of using the ANSYS numerical model is different from that constructed in MATLAB. Thus, the parameters adopted in the genetic algorithm were different. The table [4] shows the adopted values.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>ANSYS</th>
<th>MATLAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>POPULATION SIZE</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>NUMBER OF GENERATIONS</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>CROSSING RATE</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>MUTATION RATE</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>MIGRATION RATE</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>ELITISM RATE</td>
<td>0.1</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Different from the work of Oliveira (2017), in this dissertation, the number of generations was kept slightly below that one hundred times the number of variables to be identified, seeking to obtain a gain in computational time. However, the number of individuals chosen is the minimum to solve the inverse problem for the model in ANSYS® and almost double that required for the MATLAB® model. It is expected that with these values it will be possible to obtain the minimum of the objective function for both models.

4.3.3 Hybrid Algorithm

Finally, the hybrid algorithm will consist first of obtaining the solution by AG and this will serve as initial kick for the Nelder-Mead or SQP method. The choice of this order aims to unite the ability of the genetic algorithm to find global minimums with that of the NLOT to refine the search for the optimal location.

For the frequency, method optimization was applied by Nelder-Mead without using penalties to restrict the domain of the solution. This technique
allowed to obtain possible locations and depths for the crack without much difficulty. As for the other methodologies, especially for the mode, the SQP was used as NLOT, because the restrictions became necessary and the application of penalties was not desired due it introduced complexity in the implementation of the optimization technique.

As for the parameters used in the genetic algorithm, they were the same ones used in the technique that only applies GA. That is, the parameters was presented in the table [4] for each numerical model.

4.4 PHYSICAL MODEL

4.4.1 Beam

The structure tested for this dissertation is a metal beam of rectangular section with contour conditions free-free, obtained by means of elastic suspension. Some reasons for choosing this type of structure are the applicability of the structure within the industry, ease of manufacturing, the number of jobs that involved it, and the convenience of the testing and numerical modeling. The free-contour condition was chosen, due it avoids having to identify the contour effect. The table [5] presents the average dimensions and properties of the material of the specimens tested.

| TABLE 5 – PROPERTIES AND AVERAGE DIMENSIONS USED. |
|-----------------------------------------------|-----------------|-----------------|
| VALUE | UNIT   | ELASTICITY MODULE (E) | 210 GPa |
| DENSITY ($\rho$) | 7860 kg/m³ |
| POISSON COEFFICIENT ($\nu$) | 0.3 - |
| LENGTH (L) | 0.5 m |
| HEIGHT (h) | 2.50E-02 m |
| WIDHT (b) | 1.20E-02 m |

Before the identifications are made, the parameters of the material and length are optimized, so that the numerical and experimental model without cracks have the smallest difference between their natural frequencies. The optimization of the Poisson coefficient, height and depth were not performed. The
first one because his influence on the modal parameters was disregarded and
the other two because they were supplied by the manufacturer with an accuracy
of the order of 0.0001. The length does not have the same precision, because
the beam has been purchased of a larger size and has been cut, to reach a
dimension close to the desired one. In the optimization, only genetic algorithm
was used, whose parameters for characterization in MATLAB® and ANSYS®
models are shown in table [6].

<table>
<thead>
<tr>
<th></th>
<th>MATLAB</th>
<th>ANSYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>POPULATION SIZE</td>
<td>200</td>
<td>30</td>
</tr>
<tr>
<td>NUMBER OF GENERATIONS</td>
<td>150</td>
<td>50</td>
</tr>
<tr>
<td>CROSSING RATE</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>MUTATION RATE</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>MIGRATION RATE</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>ELITISM RATE</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Analyzing the table, it is noticed that both the population and the number
of generations is smaller for the characterization of the material and length in the
ANSYS models. The reason for it is in the fact that for each individual a modal
analysis was performed in the software that, even not taking long time, increased
the computational time for evaluation of the generation absurdly. Therefore, these
parameters were minimized in order to perform the optimization in a shorter time.
Only two specimens were tested. However, the position of the cracks was
different and the depth was increased until the almost complete fracture of the
structure. In the next section, the values for the cracking parameters used will be
discussed.

4.4.2 Crack

Both the first and the second beam had only one crack being positioned at
237 mm and 112.7 mm from the left end, respectively. The reason for using two
structures with different crack positions is to verify how the identification
algorithms behave and what are the precisions obtained.
About the construction of the crack, it was made through a fine cut, about 1 mm, made with the aid of a handsaw. When the tear is small, the behavior exhibited by the beam is similar of the structure with a real crack. This is because the non-linearities caused by a real crack do not impact on the natural frequencies and vibration modes obtained (SILVA AND GOMES, 1990).

As for the depth of the crack, initially the beams were tested without fissure to obtain the modal parameters of the intact structures. After this, the cracks were introduced with 1.5 mm of depth and repeated the experiment. Once this was done, the depths were increased by 1.5 mm and tested until the near breaking of the beam. Briefly, the depths ranged from 0 to 22.5 mm with a pitch of 1.5 mm.

4.5 EXPERIMENTAL PROCEDURE

4.5.1 Measurement points

To obtain the modal parameters, measurements were taken at 19 equally spaced points along the beam. These points were placed on the face where the crack began, because, during the test, the collisions were performed on the face opposite to the crack. The figures [32] and [33] show the schematic drawing of the position of the measuring points and the divisions in the two beams used. The figure [32] has only an illustrative character, and the beam and the crack are out of scale.

FIGURE 32 – SCHEMATIC DESIGN OF MEASUREMENT POINTS AND IMPACT.

![Diagram of measurement points and impact](source: the author.)
In order to reduce the measurement errors, a hybrid optimization algorithm was built into the MATLAB, which allowed varying the coordinates of these 19 points, so that the numerical modes were as close as possible to those obtained experimentally. This algorithm counted on an initial phase using GA, followed by optimization by Nelder-Mead method. The coordinates could varied only 5 mm around the starting point. The table [7] shows the parameters used in the genetic algorithm for both the MATLAB model and the ANSYS model.

<table>
<thead>
<tr>
<th>TABLE 7 – GA PARAMETERS FOR USED MODELS</th>
<th>MATLAB</th>
<th>ANSYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIZE OF POPULATION</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>NUMBER OF GENERATIONS</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>CROSSING RATE</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>MUTATION RATE</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>MIGRATION RATE</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>ELITISM RATE</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The GA values for the optimization of the coordinates of the model in ANSYS are close to that of the model in MATLAB, because the simulation was solved once, obtaining the modes of vibrating with many points. With that, the coordinates were varied between these points to find the minor error. That is, it was not necessary to perform a modal analysis within the ANSYS in each individual, which allowed using larger populations without increasing the computational expense too much.

4.5.2 Sensors and hammer used
As it searchs to compare the effect of the technique of measurement of the modal parameters on the identification, an experimental modal analysis and an operational modal analysis were performed for each simulated crack. For the EMA, the accelerometer M355B02 SN2826 and the hammer of impact 086C03 were used, both instruments belong to PCB. The measuring range of the accelerometer is from 1 to 10 kHz with a sensitivity of 10 mV / g, while the hammer measures up to ± 2224 N peak with a sensitivity of 2.25 mV / N, and both instruments are piezoelectric. The figures [34] and [35] show the hammer and the sensor attached to the structure, respectively.

FIGURE 34 – IMPACT HAMMER 086C03.

FIGURE 35 – ACCELEROMETER M355B02 SN2826 FIXED ON THE BEAM BY MAGNETIC BASE.

For the operational modal analysis, seven accelerometers were used, the first being considered as a reference and the others being displaced in the
structure to compose the mode. The impact hammer used was even used in the EMA, but it was not connected in the acquisition system, because it was not necessary to collect the applied force curve. The accelerometers were 3225F1T model of the Dytran Instruments and they had a measuring range of 1.6 to 10 kHz with a sensitivity of 10 mV / g. The attachment of them was done with wax. The figures [36] and [37] show the sensors fixed to the structure and the boxes with the cabling used.

FIGURE 36 – 3225F1T ACCELEROMETERS FIXED WITH WAX ON THE BEAM.

SOURCE: the author.

FIGURE 37 – SENSORS AND CABLING USED.

SOURCE: the author.

4.5.3 Software and data acquisition system
The data acquisition system used was Siemens LMS SCADA Mobile in both EMA and OMA. However, in the interpretation of the data obtained in the tests, the software used was different. For the EMA, the modal parameters of the experimental data were extracted with the aid of software from the measurement system itself, the LMS Test.Lab. For the OMA, the ARTeMIS Modal was used, because it is an own software for experimental modal analysis. The figures [38] and [39] show the data acquisition system and the data of an EMA test being collected.

FIGURE 38 – LMS SCADA MOBILE FOR DATA ACQUISITION.

SOURCE: the author.

FIGURE 39 – COLLECTING EMA DATA.

SOURCE: the author.
4.5.4 Assembly of tests

In the experimental modal analysis, the accelerometer was fixed to the structure with the aid of a magnetic base. The position of allocation was the point 1 of the face where the crack starts, as shown in the drawing of figure [36]. After this, five impacts were realized with the hammer in each position and the inertia, force and coherence curves were collected by the system. After all the collisions, the modal parameters were extracted with the aid of the acquisition system software, the LMS Test.Lab.

In the operational modal analysis, there were not so many channels available to run the test of all points simultaneously. Because that, the sensor of position 1 was considered as reference (being fixed in all tests), while the other sensors were two positions away from each other. After the fixation of all the sensors with wax, random impacts were carried out along the entire beam, on the opposite face of cracks and sensors. These impacts were performed within a 30 second time window. The test was repeated twice.

After these measurements, the sensors were moved, with the exception of the first one, towards the reference sensor, so that the accelerometer closest to the fixed was at one position away from it, while the others continued with two positions between them and the next. The random impact procedure of 30 seconds was repeated twice. After this, the sensors were moved again one position in the direction of the fixed one and the battery of 2 experiments of 30 seconds each was repeated. After these three test configurations, the time acceleration data were collected and taken to the ARTeMIS Modal, so that the modal parameters could be extracted. The procedure used in the EMA and the OMA is showed, in a summarized way, at the figures [40] and [41], respectively.
In the tests of experimental modal analysis, the natural frequencies and vibrate modes of the beam with a crack were extracted, as well as in the operational modal analysis. However, in OMA, the parameters were extracted using two different estimators: the Stochastic Subspace Identification (SSI) and the Enhanced Frequency Domain Decomposition (EFDD). The SSI is an algorithm subspace-based algorithm, whereas the EFDD is based on a system
of a single degree of freedom (SDOF). The completely mathematical embasement behind the technique is best explained in its own bibliography. Chauhan's article (2015) can be cited as a brief review. Therefore, the OMA tests provided two sets of natural frequencies and modes of vibration to be analyzed, one using SSI estimator and another by EFDD.

In the next chapter, it will analyze the performance of each identification algorithm developed in the characterization of cracks. They will have as input parameters the natural frequencies and vibration modes obtained by EMA, OMA with SSI and OMA estimator with EFD.
5 RESULTS AND DISCUSSIONS

In this chapter, it will discuss the performance of each algorithm, peculiarities, advantages and disadvantages, as well as the computational time of each one. In order to make the analysis more organized, the data for each method of identification will be compiled separately.

The order in which the methods will be presented is the same one adopted in chapter 3. That is, it starts with the analysis of the wavelet method, followed by analysis of the combined, graphical, frequency and mode methods.

5.1 RESULTS: WAVELET METHOD

As previously explained, it evaluated eight wavelets with ten levels of decomposition, as well as the derivative of the curves of the coefficients obtained in each level.

To define whether a wavelet or derivative is capable of identifying the crack and if it is reliable, the errors obtained must be similar for the two beams. That is, if the differences between the actual and numerical crack of beam 1 is around 10%, those of the other beam should have values close to 10% as well. Remember that as cracks are positioned in different places, some modes may have better accuracy than others.

Due to the number of filters evaluated, it was necessary to interpolate the data obtained experimentally. Thus, the 19 points measured in each mode were interpolated to obtain 3800, 7600, 11400 and 15200 points in each mode. It is expected that the greater the number of interpolation points the better the accuracy in position identification. The data interpolation was performed for the four modes measured.

In order to ensure a better work organization, the results of each family used will be presented separately, as well as the gain obtained by the application of the numerical derivative in the coefficient curves of the wavelet transform. After that, the use of each wavelet function will be compared, with the particularities, advantages and disadvantages.
5.1.1 Daubechies

In this family, the efficiency of identification was analyzed when using $db_1$ and $db_2$. It was used ten filters, and the number of points of the mode ranged from 3800 to 15200, with step 3800. In order to look for some indication of variation in the curve, the applicability of the numerical derivative of the curves obtained by the WT was also studied.

About the results, in summary, using the numerical derivative of the coefficient curve of $db_1$ guarantees errors less than 10%. It is possible to obtain even better precisions depending on the mode, number of points and filter used. In the case of $db_2$, analyzing the WT coefficient curves guarantees an accuracy of 10 to 20%, which can be reduced to 10% depending on the combination of parameters used. Finally, the analysis of the WT curves for $db_1$ and numerical derivatives in $db_2$ did not guarantee a good identification of the fissure.

In the analyzes, it was given preference to the results of the identifications with OMA-SSI measurements, because these guaranteed better identification efficiencies than when using EMA-IMP. In addition, the results of OMA-SSI and OMA-EFD were similar.

5.1.2 Coiflets

In this group, it analyzed only the $coif1$, but also using ten filters in the decomposition by the WT. The numerical derivative was also applied.

The results indicated that the $coif1$ is not a good wavelet to be used in the identification of cracks, either through the coefficient curves or with the numerical derivatives.

5.1.3 Symlets

From this group, two functions, $sym2$ and $sym4$, were used, included their numerical derivatives curves. As they are almost symmetrical to those formulated by Daubechies, it was expected a similar behavior.

After the tests, it was noticed that only the $sym2$ presented interesting results, having similarities to the errors of the $db2$. However, $sym4$ and the
derivatives of \textit{sym2} and \textit{sym4} presented errors too high, i.e., greater than 20\%, in some combinations of filters and number of interpolation points. The reason for this is that the identification process leads to a characteristic of the curve and not the position of the fissure.

5.1.4 Discrete by Meyer

This wavelet is an adaptation of Meyer's. Moreover, as the Coiflets, the identification with the curves of \textit{dmey} or with the derivative of them is not reliable and more studies are necessary to know what it is being identified when it is analyzed the peaks and valleys.

5.1.5 Biortogonal and reverse biortogonal

From this group, it studied \textit{bior1.5} and \textit{rbio1.1}. The first one is an orthogonal pair and the second is a pair of orthogonal reversals. In this family, two wavelet functions are used, one for synthesis and another for analysis, in order to join the best properties of each one. The numerical derivative was also studied.

About the results, in summary, only the derivative applied to the WT coefficients with \textit{rbio1.1} can be used in the identification, since \textit{bior1.5}, the derivative of it and the \textit{rbio1.1} did not obtain the fissure location, since they did not emphasize the crack effect in the calculated curves.

5.1.6 Comparison between wavelets

Analyzing the errors and behaviors of each wavelet, only the derivative of \textit{db1}, \textit{db2}, \textit{sym2} (because it is symmetrical to \textit{db2}) and the derivative of \textit{rbio1.1} can be used in the identification of the crack. Comparing all of them, it is possible to notice that the most interesting mode to monitor is the third one, with the exception of in \textit{db1} and the derivative of \textit{rbio1.1}, for beam 2, because, in them, the best mode is the second, which guarantees small errors for low crack depth.

About the precision, the most accurate is the derivative of \textit{rbio1.1}, followed by the derivative of \textit{db1} and the direct use of \textit{db2} and \textit{sym2}, i.e., without numerical
derivative in both. This ranking is more qualitative, and it based on a superficial comparison of the graphs, since the precision is linked to the filter used and number of points.

In addition, using 3800 points in the interpolation and decomposition with only one filter already ensured errors less than 10% or 5% (mostly of the cases) in the identification. The only exception was for the derivative of \( db1 \), since for 15200 points, it became more interesting to use 10 filters to ensure that in the 2nd mode the errors were less than 10%.

Finally, in terms of the computational time consumed, there are no significant differences between the wavelets or derivatives, all having a low cost, in the order of a few minutes. The time tends to increase with the number of filters analyzed and points in the interpolation, but nothing very serious.

5.2 RESULTS: COMBINED METHOD

Unlike the wavelet method, for the combined methodology it is necessary to define a kind of crackled element to simulate the fissure. Therefore, it is necessary to assign values for the properties of the material, that is, for the modulus of elasticity and density. They, along with the length of the beams, were obtained from an optimization process.

This process consists in minimizing the errors between the natural frequencies of the intact numerical and experimental beams using genetic algorithms. The GA input parameters were previously provided.

The tables [8] and [9] show the values of modulus of elasticity, density and length optimized for each beam and data acquisition technique used. The experimental modal analysis is denoted by IMP (or EMA-IMP), while the operational modal analyzes with EFD and SSI criteria are referenced as EFD (or OMA-EFD) and SSI (or OMA-SSI), respectively.

<table>
<thead>
<tr>
<th>MATLAB</th>
<th>V1EFD</th>
<th>V1IMP</th>
<th>V1SSI</th>
<th>V2EFD</th>
<th>V2IMP</th>
<th>V2SSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(Pa)</td>
<td>2.0201E+11</td>
<td>2.0958E+11</td>
<td>2.0426E+11</td>
<td>2.0803E+11</td>
<td>2.0938E+11</td>
<td>2.0712E+11</td>
</tr>
<tr>
<td>( p(\text{kg/m}^3) )</td>
<td>7771.5454</td>
<td>7719.3622</td>
<td>7771.0654</td>
<td>7763.5044</td>
<td>7703.8543</td>
<td>7756.9394</td>
</tr>
<tr>
<td>L(m)</td>
<td>0.4951</td>
<td>0.5023</td>
<td>0.4966</td>
<td>0.4983</td>
<td>0.5013</td>
<td>0.5013</td>
</tr>
</tbody>
</table>
TABLE 9 – OPTIMIZED PARAMETERS FOR THE ANSYS MODEL.

<table>
<thead>
<tr>
<th>ANSYS</th>
<th>V1EFD</th>
<th>V1IMP</th>
<th>V1SSI</th>
<th>V2EFD</th>
<th>V2IMP</th>
<th>V2SSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(Pa)</td>
<td>2.0279E+11</td>
<td>2.1000E+11</td>
<td>2.0249E+11</td>
<td>2.0523E+11</td>
<td>2.1000E+11</td>
<td>2.0704E+11</td>
</tr>
<tr>
<td>ρ (kg/m³)</td>
<td>7805.2928</td>
<td>7735.5035</td>
<td>7734.9124</td>
<td>7821.9765</td>
<td>7748.9306</td>
<td>7784.6056</td>
</tr>
<tr>
<td>L(m)</td>
<td>0.4943</td>
<td>0.5015</td>
<td>0.4953</td>
<td>0.4949</td>
<td>0.5002</td>
<td>0.4965</td>
</tr>
</tbody>
</table>

The optimization of these properties was performed for the models constructed in MATLAB and ANSYS, but for the combined method only the first one is used, since the cracked element follows the formulation of the MATLAB model.

In addition, the measurement coordinates of the mode displacements were also optimized, in order to minimize errors between numerical and optimized models. The algorithm for their optimization is a hybrid between GA and the Nelder-Mead method. The parameters used were previously reported. The new coordinates obtained are shown in the tables [10] and [11].

TABLE 10 – OPTIMIZED COORDINATES FOR THE MATLAB MODEL, IN METERS.

<table>
<thead>
<tr>
<th>MATLAB</th>
<th>ORIGINAL</th>
<th>V1EFD</th>
<th>V1IMP</th>
<th>V1SSI</th>
<th>V2EFD</th>
<th>V2IMP</th>
<th>V2SSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.0298</td>
<td>0.0248</td>
<td>0.0248</td>
<td>0.0248</td>
<td>0.0248</td>
<td>0.0334</td>
<td>0.0248</td>
</tr>
<tr>
<td>X2</td>
<td>0.0548</td>
<td>0.0498</td>
<td>0.0498</td>
<td>0.0498</td>
<td>0.0507</td>
<td>0.0543</td>
<td>0.0506</td>
</tr>
<tr>
<td>X3</td>
<td>0.0798</td>
<td>0.0748</td>
<td>0.0748</td>
<td>0.0751</td>
<td>0.0748</td>
<td>0.0832</td>
<td>0.0748</td>
</tr>
<tr>
<td>X4</td>
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<td>0.0998</td>
<td>0.1017</td>
<td>0.0998</td>
<td>0.0998</td>
<td>0.1074</td>
<td>0.0998</td>
</tr>
<tr>
<td>X5</td>
<td>0.1298</td>
<td>0.1254</td>
<td>0.1301</td>
<td>0.1257</td>
<td>0.1249</td>
<td>0.1290</td>
<td>0.1248</td>
</tr>
<tr>
<td>X6</td>
<td>0.1548</td>
<td>0.1498</td>
<td>0.1498</td>
<td>0.1498</td>
<td>0.1498</td>
<td>0.1552</td>
<td>0.1498</td>
</tr>
<tr>
<td>X7</td>
<td>0.1797</td>
<td>0.1747</td>
<td>0.1747</td>
<td>0.1747</td>
<td>0.1747</td>
<td>0.1817</td>
<td>0.1747</td>
</tr>
<tr>
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<td>0.2048</td>
<td>0.1998</td>
<td>0.2011</td>
<td>0.1998</td>
<td>0.1998</td>
<td>0.2055</td>
<td>0.1998</td>
</tr>
<tr>
<td>X9</td>
<td>0.2297</td>
<td>0.2247</td>
<td>0.2262</td>
<td>0.2247</td>
<td>0.2247</td>
<td>0.2324</td>
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</tr>
<tr>
<td>X10</td>
<td>0.2548</td>
<td>0.2498</td>
<td>0.2551</td>
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<td>0.2498</td>
<td>0.2567</td>
<td>0.2498</td>
</tr>
<tr>
<td>X11</td>
<td>0.2798</td>
<td>0.2748</td>
<td>0.2830</td>
<td>0.2748</td>
<td>0.2748</td>
<td>0.2828</td>
<td>0.2748</td>
</tr>
<tr>
<td>X12</td>
<td>0.3048</td>
<td>0.2998</td>
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<td>0.2998</td>
<td>0.3079</td>
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</tr>
<tr>
<td>X13</td>
<td>0.3297</td>
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</tr>
<tr>
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<td>0.3748</td>
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<td>0.3748</td>
</tr>
<tr>
<td>X16</td>
<td>0.4048</td>
<td>0.3998</td>
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<td>0.3998</td>
</tr>
<tr>
<td>X17</td>
<td>0.4298</td>
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<td>0.4248</td>
<td>0.4248</td>
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</tr>
<tr>
<td>X18</td>
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<td>0.4553</td>
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</tr>
<tr>
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<td>0.4797</td>
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<td>0.4747</td>
<td>0.4747</td>
<td>0.4789</td>
<td>0.4747</td>
</tr>
</tbody>
</table>
Having the coordinates, the modulus of elasticity, the optimal density and length it is possible to apply the combined methodology. Unlike the graphing, frequency and mode methods, there is necessity of a numerical model for the entire beam, only for the crack. For the fissure an Euler-Bernoulli beam element, whose stiffness reduces with crack growth, is used.

As explained in the previous chapter, three different techniques were used to obtain the minimum for the identification problem. That is, only GA, a hybrid of GA and SPQ and only SPQ. The parameters of each were explained previously. The position identification errors for the four modes of the two beams using the OMA-EFD measurement are shown in the figures [42] and [43]. The mode data were not interpolated and the different optimization techniques are compared.

<table>
<thead>
<tr>
<th>ANSYS</th>
<th>ORIGINAL</th>
<th>V1EFD</th>
<th>V1IMP</th>
<th>V1SSI</th>
<th>V2EFD</th>
<th>V2IMP</th>
<th>V2SSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.0298</td>
<td>0.0248</td>
<td>0.0248</td>
<td>0.0248</td>
<td>0.0248</td>
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<td>0.0248</td>
</tr>
<tr>
<td>X2</td>
<td>0.0548</td>
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<td>0.0498</td>
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<td>0.0506</td>
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</tr>
<tr>
<td>X3</td>
<td>0.0798</td>
<td>0.0749</td>
<td>0.0748</td>
<td>0.0755</td>
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<td>0.0833</td>
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</tr>
<tr>
<td>X4</td>
<td>0.1048</td>
<td>0.0998</td>
<td>0.1016</td>
<td>0.0998</td>
<td>0.0998</td>
<td>0.1052</td>
<td>0.0998</td>
</tr>
<tr>
<td>X5</td>
<td>0.1298</td>
<td>0.1256</td>
<td>0.1303</td>
<td>0.1257</td>
<td>0.1248</td>
<td>0.1291</td>
<td>0.1248</td>
</tr>
<tr>
<td>X6</td>
<td>0.1548</td>
<td>0.1498</td>
<td>0.1499</td>
<td>0.1498</td>
<td>0.1498</td>
<td>0.1552</td>
<td>0.1499</td>
</tr>
<tr>
<td>X7</td>
<td>0.1797</td>
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<td>0.1748</td>
</tr>
<tr>
<td>X8</td>
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<td>0.1998</td>
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<td>0.1998</td>
<td>0.1998</td>
<td>0.2050</td>
<td>0.1998</td>
</tr>
<tr>
<td>X9</td>
<td>0.2297</td>
<td>0.2248</td>
<td>0.2257</td>
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First, even with crack growth, there is no gain in accuracy of identification in any of the techniques. In addition, there is a clear difference between the accuracy for the fissure location between of the two beams. This disparity is
evident when using only NLOT (FMINS). The errors for beam 1 in the FMINS is approximately 18% for all modes and cracks, while for beam 2 this error is 7.4%.

That is, it is likely that what is being identified is a surface characteristic of the objective function and not the crack. In order to remedy this doubt, the figures [44] and [45] show the objective function surfaces of the crack with 12 mm depth for the two beams using the OMA-EFD measurement and without interpolating the mode data.

FIGURE 44 – SURFACE OF THE OBJECTIVE FUNCTION - BEAM 1 – OMA-EFD – A=12mm

SOURCE: the author.
FIGURE 45 – SURFACE OF THE OBJECTIVE FUNCTION - BEAM 2 – OMA-EFD – A=12mm

As expected, the minima of the objective functions are not close to the coordinates of the crack. For the two beams, the minimums are preferably located in the middle of the structure and with depths close to 0, even when the fissure having almost 50% of depth. That is, the objective function does not allow obtaining the parameters of location and severity of the crack.

The reason for this may lie in the fact that just the coordinate displacement data was collected, not the rotation data. That is, only one degree of freedom was collected for each node. However, in the matrix of the cracked element, independent of the Euler-Bernoulli or Timoshenko formulation, it has two degrees of freedom per node. Thus, information is lacking so that identification can be possible.

Interpolating the mode data also does not improve accuracy, as it only imparts a smoothed character to the surfaces of the objective functions, but does not change the position of the minima.

In summary, the combined method is not a good tool for crack identification, because the minimum of the objective function does not coincide or relate with to the parameters of the fissure.
5.3 RESULTS: GRAPHICAL METHOD

Unlike previous methods, this requires a numerical model for the entire structure, not just for a cracked region. It is expected that the model whose numerical natural frequencies best approximate the experimental ones will perform better in identification. That is, the model that best describes the actual behavior of the structure will lead to minor errors in solving the inverse problem. Therefore, before discussing the obtaining of the crack parameters (position and depth), it is interesting to solve the problem directly for both models used. That is, from the location and depth of the fissure, the natural frequencies are obtained and these are compared with those collected experimentally.

After the resolution of the direct problem, it noticed that the model in ANSYS has a much greater precision than the one constructed in MATLAB, regardless of the mode or measurement technique used. Possible reason for this may be the fact that the model in ANSYS uses an area element in resolution. In addition, the element uses quadratic interpolation, improving accuracy in compare to the MATLAB model, whose interpolation is linear. Therefore, it is expected that the ANSYS model will allow to obtain smaller errors in the identifications than the MATLAB model, because it is able to simulate the behavior of the cracked structure more precisely.

In order to ensure that the presentation of the results found is done in an organized way, they will be separated by the numerical model used. Within each section, it will be addressed how each combination of mode curves impacts the accuracy, as well as the advantages and disadvantages of using this methodology.

5.3.1 MATLAB Model

As previously explained, this model consists of using beam elements with Timoshenko formulation for the intact part of the structure and Euler-Bernoulli for the element where the crack is located. It used 150 elements to construct the mesh and one of them is an Euler-Bernoulli type. The crack is modeled as a combination of the torsional spring stiffness with the loss of stiffness of the element where the crack is located. No effects of failure on the adjacent elements
were considered. The fissure had the location varying from 10 to 90% of the length of the beam with step of 1%, and depth ranging from 0 to 100% of the section height and step of 1%.

The figures [46] and [47] show the overlapping of the frequency surfaces of beam 1 and 2 with a 1.5mm and 4.5mm depth crack and data acquisition via EMA-IMP (experimental modal analysis).

FIGURE 46 – OVERLAPPING – BEAM 1 – EMA-IMP – A=1.5mm AND A=4.5mm.

SOURCE: the author.
It is possible to note that, besides the curves do not intersect accurately in the crack coordinates in any of these cases, there is more than one point where the modes intersect. Thus, the identification of each fissure can provide more than one possible location. As in the wavelet analysis, two sites were considered for cracking position (or four if the structure is symmetrical), in the graphical method, the first three intersections will be considered.

To determine the accuracy of the methodology, it will be compared the parameters identified by these points with the real ones and defined as error in the characterization the smallest of them. The choice of the first three crossing points is to ensure that the fissure is identified. It is worth mentioning that in the experiments performed, the crack was located in the 0 to 0.5 range of the beam length, being the choice of the first three sufficient. In a real application for a symmetric structure, the points chosen should be six, being the first three and the last three intersections.

The position error is calculated by the equation [46] and the depth error is obtained by the equation [47].

\[
E_a(\%) = \left| \frac{a_t - a}{h} \right| \times 100\%
\]
where $a_i$ is the identificated depth, $a$ is the depth of the actual crack and $h$ is the height of the beam.

In the figures [48] to [55] are shown the comparisons between the accuracy of the possible combinations of the curves in the identification of cracks from 1.5mm to 12mm depth for the two beams, using data measured by EMA-IMP.

FIGURE 48 – POSITION ERROR USING ONLY 2 MODES – BEAM 1 – EMA-IMP – A UNTIL 12mm.

SOURCE: the author.
FIGURE 49 – POSITION ERROR USING 3 OR 4 MODES – BEAM 1 – EMA-IMP – A UNTIL 12mm.

FIGURE 50 – DEPTH ERROR USING ONLY 2 MODES – BEAM 1 – EMA-IMP – A UNTIL 12mm.

SOURCE: the author.
FIGURE 51 – DEPTH ERROR USING 3 OR 4 MODES – BEAM 1 – EMA-IMP – A UNTIL 12mm.

SOURCE: the author.

FIGURE 52 – POSITION ERROR USING ONLY 2 MODES – BEAM 2 – EMA-IMP – A UNTIL 12mm.

SOURCE: the author.
FIGURE 53 – POSITION ERROR USING 3 OR 4 MODES – BEAM 2 – EMA-IMP – A UNTIL 12mm.

FIGURE 54 – DEPTH ERROR USING ONLY 2 MODES – BEAM 2 – EMA-IMP – A UNTIL 12mm.
For the identification of the depth, for beam 2, the errors grow up to 6mm deep and tend to decrease slowly after this point. As for beam 1, the errors tend to increase until the crack of 9mm, decreasing slowly. However, for this beam, the identification of fissures of 3mm and 4.5mm had better precisions than of other cracks. Another point is that the results between the different combinations are close, to most of the cracks in the two structures tested. This behavior is clearer for the second beam.

Still on the depth accuracy, analyzing the use of only two natural frequencies, it is possible to notice that for small cracks the best combination is the first and second mode for both beams, followed by the first and third for beam 1 and first and fourth for beam 2. The use of more modes in characterizing the depth of the fissure did not guarantee an increase in accuracy.

Regarding the identification of the location, the pair of first and fourth leads, for the most cases, to the worst results for beam 1. The possible reason for that is the crack is close to the nodes of these two modes. In the second structure the pair of lesser precision is of the second and third modes. The cause for that is uncertain, because the worst combination should be the one that uses the first and third modes, due to the proximity of the crack of the modal node in this case.
For combinations with more than two modes, the gain in accuracy is significant, with errors less than 5% for location of all cracks greater than 3mm for beam 2 and than 7.5mm for beam 1. And, in smaller cracks than these depths, the method remains accurate, and just only few combinations lead to errors greater than 10%. For a real application, it is recommended to use the first four modes, because they lead to errors less than 5% for cracks greater than 10% of beam height and errors less than 10% for cracks with a depth of up to 10% of structure height.

In the applied acquisition technique, using modal operational analysis ensured better precision in position and depth identification. The errors assumed values less than 1%, depending on the combination of modes used. The figures [56] and [57] show the overlapping curves for 1.5mm and 4.5mm cracks for both beams using OMA-SSI measurement.

![FIGURE 56 – OVERLAPPING – BEAM 1 – OMA-SSI – A=1.5mm UNTIL A=4.5mm.](source: the author.)
FIGURE 57 – OVERLAPPING – BEAM 2 – OMA-SSI – A=1.5mm UNTIL A=4.5mm.

Analyzing the graphs, it is remarkable the difference in the accuracy of the measurement location that uses experimental modal analysis of the one that applies operational modal analysis. The reasons for this may be the experimental procedure, which allowed a better accuracy of the natural frequencies in the OMA than in the EMA, or the optimizations in the material and length of the beams. They may have allowed the EMA frequency surfaces to deviate further from those obtained by the OMA data. In the figures [58] to [65] are shown the precisions of the different combinations of modes used in the identification of cracks from 1.5mm to 12mm for both beams.
FIGURE 58 – POSITION ERROR USING ONLY 2 MODES – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 59 – POSITION ERROR USING 3 OR 4 MODES – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 60 – DEPTH ERROR USING ONLY 2 MODES – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: The author.

FIGURE 61 – DEPTH ERROR USING 3 OR 4 MODES – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: The author.
FIGURE 62 – POSITION ERROR USING ONLY 2 MODES – BEAM 2 – OMA-SSI – A UNTIL 12mm.

FIGURE 63 – POSITION ERROR USING 3 OR 4 MODES – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
In terms of depth, there was no gain in accuracy, and the errors for all combinations were close, with a tendency to increase until a maximum in the identification a depth of 9mm and reduce slowly. This same behavior occurred with the identification made from the EMA data, but with different depth values.

In relation to obtaining the location of the crack, the gain is visible between the measurement procedures. For the beam 1, all combinations of two or more
modes led to errors of less than 5%, except the pairs of the first and third, second and fourth and third and fourth, and this last only for the 9mm crack. For the second structure, only a few combinations at some depths led to errors of more than 5%, but still less than 10% with a majority of them less than 3%.

But the most impressive gain is in groups of more than two modes. For cracks with depths greater than 4.5mm (20% of beam height), all combinations led to errors of less than 1.2%. For cracks smaller than this depth the errors were smaller than 4%, except for the combination of the first, second and fourth mode for 3mm crack (13.3% of beam height) in the second structure. As the crack increases, the errors of all pairs approach the same value, being it less than 0.5%, for beam 2, whereas, for the beam 1, the values oscillate, but still less than 1.2%. These behaviors are similar to those found in the identification using the EMA-IMP data, but with worse accuracies.

About the computational time consumed, for the construction of all the necessary surfaces, the maximum time spent was of 6 minutes, being the identification realized in less than one. It is worth mentioning that these surfaces only need to be generated once, and the parameters of all fissures can be obtained by overlapping of different cuts of these maps.

That is, the graphic method guarantees a good precision in the identification of the fissure, and the best combinations of two curves are of the first and second or of the first and fourth mode. But in a real application, it is recommended to use the three or four first modes, because they led for minor errors, thus increasing reliability. In terms of the experimental procedure, OMA-SSI presented better results than OMA-EFD and EMA-IMP. The reasons, as said, can be the experimental precision, the criterion of identification of the natural frequency and optimization of the mechanical properties and of the structure size.

5.3.2 ANSYS Model

Unlike the model constructed in MATLAB, this uses two-dimensional elements with quadratic interpolation. The finite element mesh is constructed with PLANE183 elements, whose operation was explained in item 3.2.4. There is no presence of a special element for the crack, being it modeled as an absence of area. The figures [66], [67], [68] and [69] show the overlapping curves in the
identification of cracks of 1.5mm and 3mm for the two beams using EMA-IMP and OMA-SSI measurements. The purpose of presenting the graphic overlaps for these two procedures together is to demonstrate how much the acquisition technique influences the identification.

FIGURE 66 – OVERLAPPING – BEAM 1 – EMA-IMP – A=1.5mm AND A=4.5mm.

SOURCE: the author.

FIGURE 67 – OVERLAPPING – BEAM 2 – EMA-IMP – A=1.5mm AND A=4.5mm.

SOURCE: the author.
As in the MATLAB model, in the ANSYS model, the difference in precision overlapping curves is clear, when OMA is used instead of EMA. In the data of operational modal analysis, the curves intersect closer to the fissure coordinate, different from the data of experimental modal analysis. As stated earlier, the reasons may be precision in measurement, better controlled test or the effect of optimization of properties.
Unlike from the results of the MATLAB model, when using the model implemented in ANSYS, the intersection point has both location and depth close to the real ones. Before discussing the precisions found for each combination, in the figures [70] to [77] are shown the identification graphs for cracks up to 12mm in both beams with measurement of natural frequencies through OMA-SSI. The results of EMA-IMP and OMA-EFD will not be shown, because, in the first one, worse values of precision are obtained, while in the second, errors are close to OMA-SSI.

FIGURE 70 – POSITION ERROR USING ONLY 2 MODES – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 71 – POSITION ERROR USING 3 OR 4 MODES – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 72 – DEPTH ERROR USING ONLY 2 MODES – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 73 – DEPTH ERROR USING 3 OR 4 MODES – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 74 – POSITION ERROR USING ONLY 2 MODES – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 75 – POSITION ERROR USING 3 OR 4 MODES – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 76 – DEPTH ERROR USING ONLY 2 MODES – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
For depth identification, it is possible to note the superior precision of the model constructed in ANSYS compared to the model implemented in MATLAB. This was already expected, since the points of intersection are closer to the actual fissure parameters in the model in ANSYS. A common point between the models is about the behavior of the solution, because the tendency to increase of the errors as the crack grows repeats for this model, but in a much less pronounced way.

In quantitative terms, it is possible to note that, for beam 1, the errors combining two curves are less than 5%, except for the pairs of the second and fourth and first and fourth modes. For the beam 2, the errors for the combination of two curves were less than 2%, except for the pair that uses the third and fourth modes. The accuracy becomes even better when using more modes in the combination passing for errors less than 1.8% for both beams.

Regarding the location, it is noticeable that the errors are very similar between the models. That is, the gain in accuracy for both beams is not more than 5% for most combinations. However, it is worth mentioning the 1.5mm crack that had errors less than 1% using three to four modes for the two structures, except for the combination of first, second and third mode.
As seen in the results, the use of the ANSYS model ensured excellent identification results, not only of the position, but also in the depth of the crack. This behavior was more accentuated when used by operational modal analysis. However, about the computational time for the construction of frequency surfaces, the ANSYS model was much worse than MATLAB. That is, to generate the frequency variation graphs for the four modes, it was required an average of 7 hours, a time much longer than the 6 minutes that the MATLAB model used. Already in the identification, it took a few seconds, just as it was for the construction in MATLAB. That is, even though the graphics require a high computational cost, still the method is useful, since they have to be built only once. In a real application, the maps can be generated, and, as the fissures arise, they can be identified in a few seconds.

5.4 RESULTS: FREQUENCY METHOD

As explained above, in addition to the numerical model for the structure, this methodology requires an optimization technique to obtain fissure parameters. In general terms, the frequency method can be seen as an evolution of the graphical method. That is, it seeks to find the parameters of the crack that guarantee the same difference in the frequencies measured experimentally, without the need to evaluate all possible combinations of depth and position of the crack.

Thus, as in the previous method, it is expected that the ANSYS model presents a better precision than the one constructed in MATLAB, due the first one describes the behavior of the structure better than the second. In addition, it is expected that the hybrid algorithm presents better results than the technique that it uses only the genetic algorithm or only the nonlinear optimization technique. This is because the hybrid combines the advantages of the two other techniques.

Regarding the method variant, the program allowed comparing the experimental and numerical frequencies directly. Besides that, it is possible to compare the differences between the natural frequencies of intact and cracked numerical structure with the variations between the natural frequencies of the intact and cracked experimental structure. It is expected that the variant using the
differences has a better accuracy than the one that uses the frequencies directly. The reason for this is that, in the difference variant, the variations are normalized by the frequency of the structure in each mode, decreasing numerical errors of the method.

In order to ensure a better presentation of the results, they will be divided according to the numerical model used and variant chosen. In addition, it will focus on the errors obtained by the frequency data obtained by the operational modal analysis with SSI estimator. The reason for this is that the accuracies identified by OMA-SSI data are better than EMA-IMP and similar to the OMA-EFD.

5.4.1 MATLAB Model – Frequencies applied directly

The presence of cracks in structures changes the modal parameters of it, especially the natural frequencies. In the variant of the frequency method that uses the absolute value of the frequencies, it will be searched the parameters of position and depth of the fissure that allow obtaining the same natural frequencies measured experimentally. As described in Narkis (1994), at least two natural frequencies are required to identify a crack. Thus, the determination of the crack parameters will only be done using at least two modes. The next figures show the identification errors for cracks from 1.5mm to 12mm depth.

In the images are presented the characterizations using the first two modes (graphic to the left), the first three (to the center) and all four frequencies acquired (to the right). The legend of the graphics refers to the optimization technique used. In the 'ALGEN' it is used only genetic algorithm, in the 'HYBRI' it is applied the hybrid algorithm between GA and NLOT and in the 'FMINS' is implemented only the technique of nonlinear optimization. In the figures [78] to [83] are shown the times required to solve the inverse problem of each fissure for each technique chosen.
FIGURE 78 – POSITION ERROR – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 79 – DEPTH ERROR – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 80 – IDENTIFICATION TIME – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 81 – POSITION ERROR – MATLAB MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
In the identification of fissure position, the method using the frequencies with the absolute value and MATLAB model presented very good results. That is, most of the errors obtained were less than 4%, some of them were less than 1%. 

**SOURCE:** the author.
The exception was the crack of 1.5mm for beam 2 when only the first two frequencies were used in the identification, because the error obtained was greater than 12%. The reason for this may be that the fissure was close to the first mode node, which may have made it difficult to measure the first natural frequency. However, this should occur for beam 1 also, since the fissure was close to the second mode node.

In the obtaining the depth, it is possible to notice an abnormal behavior in the identification. That is, the errors tend to grow with the growth of the fissure, reaching its maximum value when the depth crack is near to near 9mm and it reduce slowly after this point. Such behavior also occurred in the identification by the graphic method with MATLAB model, so, the possible cause can be the formulation of the elements used. Another point that should be highlighted is that the choice of resolution technique did not significantly affect the depth errors, especially when using more than two modes.

In terms of the optimization technique used, the hybrid algorithm presented similar results to GA and better than those obtained by NLOT. However, analyzing the graphs of the required times, it is possible to notice that there was not a significant addition of time from the hybrid to the genetic one. That is, the NLOT stage after the GA did not confer guaranteed a clear improvement of the results for any of the analyzed cracks.

As for NLOT, the results were worse or similar to that obtained by GA. However, the time required was seven times higher than that used by the genetic algorithm. In other words, in addition to the precisions that are not better than GA, the computational cost was much higher, making the application of the technique uninteresting.

About GA, besides to have presented very good results, it consumed little computational time. The figure [84] shows details of the solving process of the inverse problem for the 1.5mm crack of beam 1 using only two modes with GA technique.
Analyzing the figure [153], it is possible to realize that the algorithm did not need all generations to identify the fissure. In addition, with less than ten generations the minimum has already been located and the other iterations was useful only to confirm it. In the third graph, also of the image [153], the average distances between the individuals are presented. As a high mutation rate was chosen, the population radius remained high and well oscillating. That is, the minimum obtained can be considered the global, since it remained the same for almost half of the available generations, even with the high rate of mutation and crossover used. Thus, in a practical application, using only genetic algorithm may be enough for the resolution by frequency method with MATLAB model.

5.4.2 MATLAB Model – Differences between the frequencies

As previously explained, in this variant, the frequency differences of the numerical and experimental cracked and intact structures are compared. The variations of each mode are normalized by the frequencies of the intact numerical and experimental beams, as equated in the materials and methods chapter. Because of this normalization, the accuracies are expected to be better than
when the frequencies were used directly. In the figures [85] to [90] are shown the graphs for identification of cracks from 1.5mm to 12mm.

FIGURE 85 – POSITION ERROR – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 86 – DEPTH ERROR – MATLAB MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 87 – IDENTIFICATION TIME – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 88 – POSITION ERROR – MATLAB MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
Analyzing the graphs of errors in the identification of the depth of the fissure, it is possible to see that the precision worsens with the increase of the crack. This behavior occurred previously with the other methods that used the
MATLAB model. That is, something in the construction of the element used does not allow the depth of the crack to be well modeled.

In the characterization of the position, it is noticeable that using the frequency difference did not give an improvement in the results. That is, the errors remained good, but the values were similar to the obtained by the method variant which it was applied the frequencies directly. The results were still less than 4%, ensuring that this option is interesting to determine the fissure location.

In terms of computational time consumed, the values for resolutions by hybrid algorithm and GA remained low and similar. This underscored the possibility of the hybrid algorithm only introduced complexity into the optimization, without providing an improvement in accuracy. One point that should be highlighted is that using NLOT presented higher times than in the previous variant of the method. That is, the new objective function used was more difficult to minimize. And, in terms of precisions, the NLOT continued to present worse results than the other techniques.

In summary, for the application of the frequency method with MATLAB model, the optimization technique that presented better results was that of genetic algorithms. In terms of the method choice, the both variants of the methodology showed similar precision. In the question of experimental procedure, the operational modal analysis with SSI estimator is still recommended when compared to the EMA-IMP and the OMA-EFD, because it presents similar or better results.

5.4.3 ANSYS Model – Frequencies applied directly

As in the MATLAB model, in this model the finite element method is still applied in the resolution. However, the mesh and elements used are built in ANSYS. And, for this model, more degrees of freedom are used, as well as greater complexity.

As done previously, it will be analyzed the results of identification using the OMA-SSI measurement data. However, the results of the hybrid algorithm will not be evaluated because, as shown in the MATLAB model, this technique does not improve the precision obtained by the genetic algorithm. In the figures [91] to [96]
are shown the precisions obtained for both beams for the method variant using the frequencies directly.

FIGURE 91 – POSITION ERROR – ANSYS MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

FIGURE 92 – DEPTH ERROR – ANSYS MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.
FIGURE 93 – IDENTIFICATION TIME – ANSYS MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 94 – POSITION ERROR – ANSYS MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
The difference between the optimization techniques used was visible. Analyzing the precision graphs, it was possible to notice that the NLOT, referenced in the graph as 'FMINS', presented almost constant errors in the
position identification. As for depth, the errors grew linearly until they reached the maximum for 9mm cracks. This behavior was due to the fact that the technique could not get past the first iterations. This did the initial kicks were considered as a solution to the problem. The reason for the impossibility of correctly applying the SPQ non-linear optimization technique was bad communication between ANSYS and MATLAB. The first software was responsible for obtaining the eigenvalues and eigenvectors, that is, performing the modal analyzes. The second one was needed to apply the optimization technique. Somewhere between the end of one interaction and the beginning of the next, some information was lost.

This problem among the software was confirmed by the computational time spent, which, besides to being much smaller when compared to the genetic algorithm time, was constant for the two beams and almost all cracks. That is, the nonlinear optimization technique failed to identify the cracks for the frequency method with ANSYS model. If the use of NLOT in the minimization is still of interest, a more in-depth study of this method is necessary.

Regarding the genetic algorithm, it was possible to notice that, for the crack position, there were no significant gains in accuracy when compared to the MATLAB model identifications. It is worth noting that for the beam 2 and resolution with only two modes, cracks smaller than 4mm had worse accuracies than those obtained in the previous model. But this was solved with the addition of more modes in the calculation.

The main gain of the ANSYS model was to obtain the depth of the faults, since the errors were extremely low. That is, all of them were less than 5%, and the majority were less than 3%. However, the problem of using the model in ANSYS with genetic algorithm was in the computational cost required to identify the crack parameters. The minimum time required was 40 minutes, but, for some fissures, it was bigger than 100 minutes.

In summary, the ANSYS model in conjunction with AG using the values of the frequencies directly led to similar location accuracies as the MATLAB model. However, the errors for the depth were much smaller, this being the gain of the model. If computational time is not an obstacle, ANSYS is highly recommended for crack identification.
5.4.4 ANSYS Model – Differences between frequencies

As it was done for the previous model, in this was also introduced the variant of the methodology that uses the value of the difference between frequencies. The procedure was the same as previously used, just with other structure model. For this variant, the hybrid algorithm was not used in the resolution too, for reasons previously presented. The figures [97] to [102] show the position and depth identification for the cracks in the two beams.

FIGURE 97 – POSITION ERROR – ANSYS MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 98 – DEPTH ERROR – ANSYS MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 99 – IDENTIFICATION TIME – ANSYS MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 100 – POSITION ERROR – ANSYS MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 101 – DEPTH ERROR – ANSYS MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
The non-linear optimization technique was not able to solve the inverse problem satisfactorily for any crack, number of modes used or beam analyzed. The reason for this is communication between ANSYS and MATLAB, as explained previously.

The depth errors were similar to those obtained in the other frequency method variant. However, for position accuracies, the results were better when using the difference between frequencies in the resolution. This gain was more prominent when more than two modes were used in the calculation, being all of them less than 2% and the majority less than 1%.

The computational cost required for identification continued being high, even higher than in the other variant of the method. One point that must be explained is the difference between times spent in identifying with two modes, for the beam 2, of the other computational expenses. The reason for this is that, during the resolution of the problem, computers with different performances were used, in order to speed up the calculation. This ended up influencing the result, but, even so, it was possible to get a sense of the amount of time needed.

Comparing the results of the two variants of the frequency method and the implemented models, it is possible to affirm that using the differences between
frequencies allowed obtaining better precision in both depth and location. The only constraint was on the computational cost expended by the ANSYS models in the crack identification, due it is much larger than that required when the structure was modeled in MATLAB.

In terms of techniques, regardless of the variant of the method and the model used, the best technique was GA, whereas the NLOT was the worst. The hybrid algorithm presented similar results to the genetic ones, when it worked, offering no resolution improvements.

5.5 RESULTS: MODE METHOD

In the chapter in which this method was proposed, it were suggested four options for the objective function. In them, the possibility of interpolating or not the data collected was considered. In addition, numerical and experimental modes could be directly compared or used the differences between eigenvectors of the intact and real structure with the variations between numerical vibration modes of the component with and without crack. Each of these four possibilities of solving the method could still contemplate different forms of data collection, optimization techniques, quantities of modes used and position of fissure.

However, during the identification of fissure parameters, it was noticed that some combinations were not possible. Some of the problems encountered were unsatisfactory communication between ANSYS and MATLAB, and the numerical errors that accumulated during the calculations. In addition, difficulties of implementation and execution of the codes were found, as well as, a high computational cost required, especially for resolutions with ANSYS model. Thus, the table [12] shows the variants of the method that were able to solve the inverse problem with the data inputs and optimization technique chosen.

<table>
<thead>
<tr>
<th>BEAM</th>
<th>TYPE TEST</th>
<th>MODEL</th>
<th>RELATION BETWEEN MODES</th>
<th>INTERPOLATION</th>
<th>METHOD OPTION</th>
<th>OPTIMIZATION TECHNIQUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>V2</td>
<td>EFD/IMP/SSI</td>
<td>MATLAB</td>
<td>ABS</td>
<td>YES</td>
<td>VAR 2</td>
<td>HYBRI</td>
</tr>
<tr>
<td>V1/V2</td>
<td>EFD/IMP/SSI</td>
<td>MATLAB</td>
<td>DIF</td>
<td>NO</td>
<td>VAR 3</td>
<td>ALGEN/ FMINS/ HYBRI</td>
</tr>
<tr>
<td>V1/V2</td>
<td>EFD/IMP/SSI</td>
<td>MATLAB</td>
<td>DIF</td>
<td>YES</td>
<td>VAR 1</td>
<td>ALGEN/ FMINS/ HYBRI</td>
</tr>
<tr>
<td>V1/V2</td>
<td>EFD/IMP</td>
<td>ANSYS</td>
<td>DIF</td>
<td>NO</td>
<td>VAR 3</td>
<td>ALGEN</td>
</tr>
</tbody>
</table>
Before discussing the table presented, an explanation is needed on the codes used. In the beam part, 'V1' refers to the beam with a crack in 237mm, while 'V2' is for the structure with a crack in 112.7mm. In the measurement, EFD and SSI are the identifications that used the operational modal analysis data with EFDD and SSI estimators, respectively. Still on the data collected, 'IMP' is for the modes obtained through experimental modal analysis. In the method option, 'VAR' refers to the variant used and each option was explained in table [2] of the Materials and Methods chapter. Finally, in the optimization technique, 'ALGEN' and 'FMINS' refer to the use of only the genetic algorithm or only the non-linear optimization technique, respectively. While 'HYBRI' is for when the hybrid algorithm was applied.

It can be seen from table [12] that it was not possible to apply interpolation in the modes and to use the differences between cracked and intact structures in the objective function, independent of the chosen model, data acquisition procedure or applied optimization technique. In addition, for the second variant of the modes method, it was only possible to apply it when using the MATLAB model, the second beam input data and the hybrid optimization algorithm. The reason for just this combination of factors was explained previously.

Still on the MATLAB model, different from what happened in the frequency method, for the methodology of the modes, it was possible to implement only the NLOT in the resolution for the first and third variant. That is, this optimization technique was able to lead to satisfactory identification results.

However, for the model in ANSYS, only the third variant of the method can be applied and only with the genetic algorithm. The reason for the NLOT or the hybrid algorithm did not work was the same for impossibility of applying these techniques in the frequency method with model in ANSYS. The nonlinear optimization technique did not show good functioning, due to communication problems between software and internal code errors.

For a better organization and presentation of the obtained results, these will be divided according to the model of the structure used and variant of the methodology approached. It will be given preference to the identifications by the operational modal analysis data with SSI estimator, when possible.
5.5.1 MATLAB Model – First variant of the method

In this variation of the methodology, the data of the modes are interpolated. The objective function compares the differences between them of the actual structure with the differences between numerical modes. It was expected, therefore, to reduce the effect of measurement noise on the identification, since both the collected modes of the intact structure and those of the cracked beam presented intrinsic measurement errors. Moreover, with the combination of modes, these errors were subtracted from each other, minimizing their influence. The equation of the objective function used is better explained in the Materials and Methods chapter.

For the MATLAB model, this variant of the method was able to identify the fissure parameters, regardless of the chosen experimental procedure or the applied optimization technique. A differential of the mode method over that of the frequency is that the fissure can be characterized using only one mode of vibration. This is because a certain variation in the eigenvector is linked to only a combination of position and depth of the crack. The figures [103] to [108] show the graphs with the errors found in the identification of the fissure parameters.

FIGURE 103 – POSITION ERROR – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 104 – DEPTH ERROR – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 105 – IDENTIFICATION TIME – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 106 – POSITION ERROR – MATLAB MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 107 – DEPTH ERROR – MATLAB MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
The precision results in the position identification were worse than in the frequency method. Possible reasons for this may have been the measurement errors present, since the data for the vibrate modes present a higher noise than the measurements of the natural frequencies. However, it was still possible to identify the position of the crack, especially for cracks larger than 4mm, since for them the error was equal to or less than 10%. In addition, the precisions tended to be better when three or four modes were used in the calculation.

As for the errors in the depth, they increased with the growth of the fissure until reaching maximum values close to the 9mm crack. This behavior was repeated in all the identification methodologies that used the MATLAB model, not repeating itself to those that modeled the structure by ANSYS. That is, this behavior was a result of the way the structure was modeled, as was explained previously in other analyzes.

In terms of the optimization technique chosen, it was noticed that all of them presented values close in the identification of position and depth. That is, any one of them was able to characterize the crack, leading to similar results. For the computational cost consumed, for the beam 2, the times were close, but, for
the cracks of the first structure, the NLOT presented times much better than the other techniques.

In addition, about the problem resolution tools, regardless of the number of modes used in the identification, the hybrid algorithm presented values extremely close to those obtained by GA. Initially, the possibility was considered that the NLOT part of the code is not working, as it happened with the ANSYS model. However, when it analyzed the graphs of the times consumed, it was noticed that the technique consumed a different time of the AG, being even bigger for the identifications of the fissures in the second beam. That is, the hybrid algorithm worked, but did not improve accuracy significantly.

In summary, the mode method using the first variation with MATLAB model was able to identify the position of fissures larger than 4mm reliably, with errors less than 10%. However, for the depth, the numerical model influenced the resolution, leading to results higher than 15%.

5.5.2 MATLAB Model – Second Variant of the Method

This variant used the interpolation of the collected vibration modes and compared the values measured experimentally with those calculated numerically. In the previous methodology option, the objective was to ensure that the variations between cracked and intact modes of the real and numerical structures were the same. Already in this variant of the method, it was sought to match the behavior of the computational model with that of the real component. Because of this, the results for this variant of the method were expected to be worse than the first, since noise was not minimized as previously done.

As explained, it was not possible to identify the fissure parameters for the two beams or to use all proposed optimization techniques. Thus, the figures [109], [110] and [111] show the results for position and depth obtained for the cracks of the second beam. The only optimization technique evaluated was that of the hybrid algorithm. The reasons for this were explained previously.
FIGURE 109 – POSITION ERROR – MATLAB MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

FIGURE 110 – DEPTH ERROR – MATLAB MODEL – BEAM 2 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
From graphs [109] and [110], it was possible to note that the errors were similar to those of the previous variant of the method. That is, the noise reduction previously proposed did not have as much effect on the results as it was expected. Different from the previous option of the methodology, in this, the cracks with worse characterizations of the position were that they had the 3mm and 4.5mm depth, and not the cracks of 1.5mm, as it happened previously. But, the behavior in the depth identification was repeated, that is, errors increased with the length of the fissure. However, it is worth noting that the worst accuracy did not occur for 9mm or close cracks, but for 1.5mm cracks. About the time required in the calculations, they remained similar to the previous variant with hybrid algorithm optimization technique.

It was not possible to apply this variant to the vibration modes of the first beam and to use other minimization techniques. Thus, the behavior of the method is unknown, when the fissure is in different position or with different procedures to obtain the solution. That is, this variant is less reliable than the previous one, even its results having similar.
5.5.3 MATLAB Model – Third variant of the method

The last variant of the methodology used the difference of modes, but did not interpolate them. As with the first option of the method, it was possible to identify the fissure, independently of the adopted experimental procedure, the position of the crack or the optimization technique chosen. The figures [112] to [117] show the accuracy results for the two beams tested.

FIGURE 112 – POSITION ERROR – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 113 – DEPTH ERROR – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
FIGURE 114 – IDENTIFICATION TIME – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.

FIGURE 115 – POSITION ERROR – MATLAB MODEL – BEAM 1 – OMA-SSI – A UNTIL 12mm.

SOURCE: the author.
For the second beam, the position and depth results were similar, but, for the first beam, they were worse. However, in relation to computational time, the third variant required less time than the first and second in the resolution. The reason may have been that, in this methodology option, the modes are not interpolated. That is, in the objective function less data is used in the minimization, decreasing the number of calculations.
Of these variants tested for the MATLAB model, none presented better results than the frequency method, being, at most, similar. That is, the mode method using the MATLAB model is less interesting for a real application, because it requires a larger input and more precise experimental procedure. In addition, the identifications do not have a significant gain in relation to the other methodologies, either in time or in precision.

5.5.4 ANSYS Model – Third variant of the method

For the numerical structure constructed in ANSYS, it was only possible to apply the third variant of the mode method. In addition, in terms of optimization techniques, just the genetic algorithms led to useful results. The figures [118] to [123] show the position and depth identification errors for the cracks of the two beams tested.

FIGURE 118 – POSITION ERROR – ANSYS MODEL – BEAM 1 – OMA-EFD – A UNTIL 12mm.

SOURCE: the author.
FIGURE 119 – DEPTH ERROR – ANSYS MODEL – BEAM 1 – OMA-EFD – A UNTIL 12mm.

SOURCE: the author.

FIGURE 120 – IDENTIFICATION TIME – ANSYS MODEL – BEAM 1 – OMA-EFD – A UNTIL 12mm.

SOURCE: the author.
FIGURE 121 – POSITION ERROR – ANSYS MODEL – BEAM 2 – OMA-EFD – A UNTIL 12mm.

SOURCE: the author.

FIGURE 122 – DEPTH ERROR – ANSYS MODEL – BEAM 2 – OMA-EFD – A UNTIL 12mm.

SOURCE: the author.
For the second beam, the identification of position and depth were better in the model in ANSYS than in MATLAB. In addition, the error in the length of the fissure oscillated around 10%, not growing with the increase of the crack.

However, for the first component, position accuracy was similar or worse when the other numerical model was used, except for identifications that used three or four modes. In addition, the depth error showed an oscillating behavior, increasing or decreasing randomly. That is, the method did not present a stable behavior for identification, since the errors for the cracks of the first beam are appreciably worse than the second structure, besides alternating without a clear reason.

In terms of time consumed, just as it was for the frequency method, in this, the characterization required a much longer time than when the structure was modeled in MATLAB. In addition, among the proposed methodologies for fissure identification using the ANSYS model, the mode method presented the highest computational cost.

Therefore, the mode methodology using ANSYS model and third variant of the method is not so interesting for practical applications, because it consumes a superior computational time and has low reliability.
5.6 RESULTS: ADVANTAGES AND DISADVANTAGES

Depending on the goal to be achieved, some identification methodologies are more effective than others. In terms of computational cost, the one that consumed the shortest time was the method called wavelet followed by graphical methods, frequency and mode, regardless of the chosen model or optimization technique.

The reason for the time of the graph method is less than that of the frequency and of the mode is in the fact that this methodology requires a high computational expense only to generate the surfaces of variation in the natural frequencies, which are calculated only once. That is, after obtaining the necessary graphics, the identification can be made in a few minutes and the growth of the fault can be monitored by monitoring the natural frequencies of the structure.

In the case of the method that uses the wavelet transform, even though it consumed the least computational time, its applicability is restricted to obtaining only the location of the crack, because the methodology does not provide useful tools to obtain its depth. This method is closer to crack detection than its identification, due it only indicates the existence of cracks, providing its positions, but without giving an idea of the severity.

When it is sought to characterize the position and depth of the crack with the lowest associated error possible, the most indicated method is the graph, followed by the frequency and mode methods. The methodology that uses the natural frequencies presents similar results to the graphical method, since it has the same foundation, but it applies techniques of optimization, instead of graphic overlap. The advantage of the frequency method on the graph lies in providing fewer possible locations for fault location, since overlays can occur at more than one point and are even more complicated for symmetric structures.

About the numerical model for the structure, the construction in MATLAB was useful to identify the position, but not to obtain the depth of the crack. The reason for this was the adopted formulation, which did not adequately represent the dynamic behavior of the structure with a fissure. Therefore, if the precision of
the depth independent of the time consumed is sought, the model in ANSYS is more indicated.

As for the optimization techniques, using only NLOT or applying hybrid algorithms did not guarantee significant improvements in any method approached. That is, the AG are sufficient to obtain the fissure parameters reliably, regardless of the model and method chosen.

Regarding the effectiveness of the implemented methods, the combined method was not useful, because the objective function did not cover all degrees of freedom of the structure, because it used only the nodal displacements and ignored the effect of the rotations. As for the method of the mode, even if it achieves some satisfactory results, the applicability of it is not interesting. The reasons for this are the complexity of implementation, the need for a larger data input, a better experimental procedure to reduce measurement noise. Moreover, this method does not provide better results than the other methodologies, adding an unnecessary complexity to the problem.

Finally, the experimental procedure that allowed better identification results was the operational modal analysis, independently of the estimator used. That is, in a real application, the OMA is more interesting than the experimental modal analysis, besides requiring less time and control of the test.
6 FINAL CONSIDERATIONS

Within the literature are numerous works with different forms of identification of cracks from the modal parameters. In this dissertation, it was tried to review the methods that used the natural frequencies and modes of vibration. In addition, different numerical models for the structure and fissure were reviewed, as well as optimization techniques and experimental procedure for data acquisition. The reason for varying so many parameters was to verify which ones presented the best performances.

In order for each proposed methodology was tested and validated, an experimental stage was performed. In it were tested two beams of rectangular section and each one had a crack in different positions. The modal parameters were collected by both operational modal analysis and experimental modal analysis. The reason was also to test the impact of different procedures of results acquisition. It was also build a data bank of cracks with different depths and positions. On the collected parameters, the methodologies were applied and the results were compared.

Among the proposed methods, the one that presented the smallest error in the identification of location and depth with low computational time was the graphic method. However, when the precision in the depth is not such important and it is only looked for precision in the crack location with low computational time, the method with the best performance was the one that used wavelet transform in the formulation. It is worth mentioning that the minimum depth identified was 1.5mm, being this value 6.67% of the height of the beam. For smaller cracks, the efficiency of the methods was not evaluated.

6.1 RECOMMENDATIONS FOR FUTURE PROJECTS

Many combinations of experimental procedures, identification method, optimization technique and numerical model were studied. Moreover, throughout the dissertation, some problems, doubts and limitations were found. Therefore, the following list suggests possible future work:

- The methodologies that monitor modal damping could be reviewed, since none of them were studied;
• The methods could be applied to structures with more cracks or with more complex geometries such as I or T profiles, pipes, real machine components, stepped beams, and others;
• The studies performed can be repeated, but with real cracks in the structure, nucleated and grown by fatigue;
• Programs for prognosis of cracks in structures with real and active cracks can be developed, obtaining not only the positions and depths of the cracks but also determining the remaining time;
• Equipment with some of the methodologies studied implemented, in order to perform the monitoring of the integrity of structures;
BIBLIOGRAPHIC REFERENCES


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